INGHAM TYPE THEOREMS AND GAP CONDITIONS

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We recall a classical theorem due to Ingham (1936)[1]:

Let $(\lambda_k)_{k\in\mathbb{Z}}$ be a given sequence of real numbers, and consider the functions of the form

$$f(t) = \sum_{k\in\mathbb{Z}} a_k e^{i\lambda_k t} \tag{1}$$

with complex coefficients (a_k).

Theorem 1. *Assume that*

$$\gamma := \inf_{j\neq k} |\lambda_j - \lambda_k| > 0 \qquad \text{gap condition,}$$

and let I be a bounded interval of length $|I| > 2\pi/\gamma$. There exist two constants $c_1, c_2 > 0$ such that

$$c_1 \sum_k |a_k|^2 \leq \int_I |f(t)|^2 \, dt \leq c_2 \sum_k |a_k|^2$$

for all square summable sequences (a_k) of complex numbers.

In this talk we briefly recall Ingham type theorems and gap conditions [2], and we describe some recent results obtained in collaboration with Daniela Sforza.

References

[1] A. E. Ingham, *Some trigonometrical inequalities with applications in the theory of series*, Math. Z. 41 (1936), 367–379.

[2] V. Komornik and P. Loreti, *Fourier Series in Control Theory*, Springer, New York 2005.

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