CARLEMAN ESTIMATES AND NULL CONTROLLABILITY PROPERTIES OF DEGENERATE PARABOLIC EQUATIONS

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Motivated by several physical problems, we are interested in controllability properties of parabolic equations degenerating at the boundary of the space domain. Typical models of such problems are:

- the following one dimensional case

$$u_t - (a(x)u_x)_x = f(x,t)\chi_{(a,b)}(x), \quad x \in (0,1), t > 0, \tag{0. 1}$$

where the control region (a,b) is a subdomain of $(0,1)$ and the function $a : [0,1] \to \mathbb{R}_+$ is continuous, of classe C^1 on $(0,1)$, and

$$a(0) = 0 = a(1);$$

- the N-dimensional case:

$$u_t - \text{div } (A(x)\nabla u) = f(x,t)\chi_\omega(x), \quad x \in \Omega, t > 0, \tag{0. 2}$$

where the control region ω is a subdomain of Ω, and the matrix $A(x)$ is definite positive for all $x \in \Omega$, and but has at least one eigenvalue equal to 0 for all $x \in \partial\Omega$.

Under some assumptions on the degeneracy, we derive new Carleman estimates for the adjoint degenerate parabolic equation, and the null controllability of the problem. Roughly speaking, if the function a does not go too fast to 0 at the boundary, the problem (0. 1) is null controllable. We also provide null controllability results for the N-dimensional problem (0. 2), when the eigenvalue goes to 0 at the boundary as some power of the distance function to the boundary.

These controlability properties of degenerate parabolic equations are closely related to the controllability properties of nondegenerate parabolic equations in unbouded demains, and some results of S. Micu and E. Zuazua (2001), L. Escauriaza, G. Seregin, V. Šverák (2004), L. Miller (2005) show that our assumptions on the degeneracy are almost optimal.

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