GLOBAL PROPERTIES OF GENERALIZED ORNSTEIN–UHLENBECK OPERATORS ON $L^p(\mathbb{R}^N, \mathbb{R}^N)$

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ABSTRACT. We show that the realization A_p of the elliptic operator $\mathcal{A}u = \mathrm{div}(Q\nabla u) + F \cdot \nabla u + Vu$ in $L^p(\mathbb{R}^N, \mathbb{R}^N)$, $p \in [1, +\infty]$, generates a strongly continuous semigroup, where $D(A_p) = \{u \in W^{2,p}(\mathbb{R}^N, \mathbb{R}^N) : F \cdot \nabla u + Vu \in L^p(\mathbb{R}^N, \mathbb{R}^N)\}$ for $p \in]1, +\infty[$. The diffusion coefficients $Q = (q_{ij})$ are uniformly elliptic and bounded together with their first order derivatives, the drift coefficients F can grow as $|x| \log |x|$, and V is a matrix valued function whose associated quadratic form is bounded. Our approach relies on the Monniaux-Prüss theorem on the sum of non commuting operators in $L^p(\mathbb{R}^N, \mathbb{R}^N)$. We also prove L^p-L^q estimates and, under somewhat stronger assumptions on the diffusion coefficients, we prove gradient L^p-estimates.

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