Inverse problem for a degenerate parabolic equation

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The main purpose of the paper is to study the possibility of identification the time-dependent coefficient in a degenerate parabolic equation.

Let $Q_T \equiv \{(x,t) : 0 < x < h, 0 < t < T\}$. Consider the following equation

$$u_t = a(t)\psi_0(t)u_{xx} + b(x,t)u_x + c(x,t)u + f(x,t), \quad (x,t) \in Q_T, \tag{1}$$

with unknown coefficient $a(t) > 0, t \in [0,T]$, initial condition

$$u(x,0) = \varphi(x), \quad x \in [0,h], \tag{2}$$

boundary conditions

$$u(0,t) = \mu_1(t), \quad u(h,t) = \mu_2(t), \quad t \in [0,T], \tag{3}$$

and overdetermination condition

$$a(t)\psi_0(t)u_x(0,t) = \mu_3(t), \quad t \in [0,T]. \tag{4}$$

It is supposed that the function $\psi_0(t)$ is given monotone increasing function, $\psi_0(t) > 0, t \in (0,T]$ and $\psi_0(0) = 0$. We considered the cases of weak and strong degenerations. The degeneration is called weak if for $t \to +0$ the expression $\int\limits_0^t \left(\int\limits_\tau^t \psi_0(\sigma)d\sigma\right)^{-1/2} d\tau$ tends to zero, and it is called strong if the named expression tends to infinity when t tends to zero.

With the aid of the Green function for the heat equation

$$u_t = a(t)\psi_0(t)u_{xx} + f(x,t), \quad (x,t) \in Q_T, \tag{5}$$

we reduce the direct problem (1)-(3) to the system of integral equations

$$u(x,t) = u_0(x,t) + \int\limits_0^t \int\limits_0^h G(x,t,\xi,\tau)(b(\xi,\tau)v(\xi,\tau) + c(\xi,\tau)u(\xi,\tau))d\xi d\tau, \tag{6}$$

$$v(x,t) = u_{0x}(x,t) + \int\limits_0^t \int\limits_0^h G_x(x,t,\xi,\tau)(b(\xi,\tau)v(\xi,\tau) + c(\xi,\tau)u(\xi,\tau))d\xi d\tau, \tag{7}$$

where $u_0(x,t)$ is solution of the heat equation with conditions (2), (3). From the condition (4) we obtain

$$a(t) = \frac{\mu_3(t)}{\psi_0(t)v(0,t)}, \quad t \in [0,T]. \tag{8}$$

Applying the Schauder fixed point theorem to the system of equations (6)-(8), we find the conditions of existence of classical solution of the problem (1)-(4). The uniqueness of solution of the problem is also established.