Multiplicity of positive solutions to nonlinear elliptic boundary value problems arising in population dynamics

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My talk is devoted to the existence and multiplicity of positive solutions to a nonlinear elliptic boundary value problem with an indefinite weight and a nonlinear boundary condition, which arises in population dynamics. The problem we consider is as follows.

$$\begin{cases} -\Delta u = \lambda(m(x)u - u^2) & \text{in } D, \\ \dfrac{\partial u}{\partial \boldsymbol{n}} = \lambda b(x) u^p & \text{on } \partial D. \end{cases} \tag{1}$$

Here D is a bounded domain of $\mathbb{R}^N$, $N \geq 2$, with smooth boundary ∂D, $\lambda > 0$ is a parameter, $m \in C^\theta(\overline{D})$ may change sign in D, $b \in C^{1+\theta}(\partial D)$ satisfies that $b \geq 0$ and $b \not\equiv 0$ on ∂D, $p > 1$ is a constant, and $\boldsymbol{n}$ is the unit exterior normal to ∂D.

It is well-known that problem (1) originates from population dynamics. The unknown function u denotes the steady state of population density of some species diffusing at rate $1/\lambda$, and $m(x)$ represents the local growth or decay rate. Our boundary condition of the form above may suggest from an ecological point of view that, if the species in the interior of D reach the boundary ∂D, then they return to the interior with some reaction according to $b(x)u^p$.

Bifurcations from the two trivial lines $(\lambda, u) = (\lambda, 0)$ and $(\lambda, u) = (0, c)$, where c is a nonnegative constant, and from infinity at $\lambda = 0$ are discussed by uses of bifurcation equations due to the Lyapunov-Schmidt procedure, super and subsolutions, and variational techniques.