MATHEMATICAL STRUCTURE FOR FOREST KINEMATIC MODEL

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1. ABSTRACT

We study the initial-boundary values problem for a parabolic-ordinary system

\[
\begin{align*}
\frac{\partial u}{\partial t} &= \beta \delta w - \gamma(v)u - fu \\
\frac{\partial v}{\partial t} &= fu - hv \\
\frac{\partial w}{\partial t} &= d \Delta w - \beta w + \alpha v \\
w &= 0 \\
u(x, 0) &= u_0(x), \ v(x, 0) = v_0(x), \ w(x, 0) = w_0(x)
\end{align*}
\]

This system has been introduced by Kuznetsov et al. [4] in order to describe the kinetics of forest from the viewpoint of the age structure. For simplicity they consider a prototype ecosystem of a mono-species and with only two age classes in a two-dimensional domain \( \Omega \).

The unknown functions \( u(x, t) \) and \( v(x, t) \) denote the tree densities of young and old age classes, respectively, at a position \( x \in \Omega \) and at time \( t \in [0, \infty) \). The third unknown function \( w(x, t) \) denotes the density of seeds in the air at \( x \in \Omega \) and \( t \in [0, \infty) \). The third equation describes the kinetics of seeds; \( d > 0 \) is a diffusion constant of seeds, and \( \alpha > 0 \) and \( \beta > 0 \) are seed production and seed deposition rates respectively. While the first and second equations describe the growth of young and old trees respectively; \( 0 < \delta \leq 1 \) is a seed establishment rate, \( \gamma(v) > 0 \) is a mortality of young trees which is allowed to depend on the old-tree density \( v \), \( f > 0 \) is an aging rate, and \( h > 0 \) is a mortality of old trees.

For \( w \), the Dirichlet boundary conditions are imposed on the boundary \( \partial \Omega \).

The initial value \((u_0, v_0, w_0)\) is taken from the space

\[
K = \{(u_0, v_0, w_0); 0 \leq u_0, \ v_0 \in L^\infty(\Omega) \text{ and } 0 \leq w_0 \in L^2(\Omega)\}.
\]

And Problem (FK) is handled in the underlying product space \( X = L^\infty(\Omega) \times L^\infty(\Omega) \times L^2(\Omega) \).

The domain \( \Omega \) is a \( C^2 \) or convex, bounded domain in \( \mathbb{R}^2 \). We assume as in [4] that the mortality of young trees is given by a square function of the form

\[
\gamma(v) = a(v - b)^2 + c,
\]

where \( a, b, c > 0 \) are positive constants. This means that the mortality takes its minimum when the old-age tree density is a specific value \( b \). As mentioned, \( d, f, h, \alpha, \beta > 0 \) are all positive constants and \( 0 < \delta \leq 1 \).

In this talk, we intend to construct a global solution to (FK) for each initial function \( U_0 \in K \) and to construct a dynamical system \((S(t), K, X)\) determined from the problem. Furthermore, we notice that \((S(t), K, X)\) enjoys a Lyapunov function and, using the
Lyapunov function, we investigate the omega limit set $\omega(U_0)$ for $U_0 \in K$. Especially, we will show a remarkable fact that, even if $U_0$ is an initial value consisting of continuous functions, $\omega(U_0)$ can contain an element $(\overline{u}, \overline{v}, \overline{w})$ which consists of some discontinuous functions.

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References


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