MATHEMATICAL STRUCTURE FOR FOREST KINEMATIC MODEL

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1. ABSTRACT

We study the initial-boundary values problem for a parabolic-ordinary system

$$\text{(FK)} \qquad \begin{cases} \dfrac{\partial u}{\partial t} = \beta\delta w - \gamma(v)u - fu & \text{in} \quad \Omega \times (0,\infty), \\ \dfrac{\partial v}{\partial t} = fu - hv & \text{in} \quad \Omega \times (0,\infty), \\ \dfrac{\partial w}{\partial t} = d\Delta w - \beta w + \alpha v & \text{in} \quad \Omega \times (0,\infty), \\ w = 0 & \text{on} \quad \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x),\ v(x,0) = v_0(x),\ w(x,0) = w_0(x) & \text{in} \quad \Omega. \end{cases}$$

This system has been introduced by Kuznetsov et al. [4] in order to describe the kinetics of forest from the viewpoint of the age structure. For simplicity they consider a prototype ecosystem of a mono-species and with only two age classes in a two-dimensional domain Ω.

The unknown functions $u(x,t)$ and $v(x,t)$ denote the tree densities of young and old age classes, respectively, at a position $x \in \Omega$ and at time $t \in [0,\infty)$. The third unknown function $w(x,t)$ denotes the density of seeds in the air at $x \in \Omega$ and $t \in [0,\infty)$. The third equation describes the kinetics of seeds; $d > 0$ is a diffusion constant of seeds, and $\alpha > 0$ and $\beta > 0$ are seed production and seed deposition rates respectively. While the first and second equations describe the growth of young and old trees respectively; $0 < \delta \le 1$ is a seed establishment rate, $\gamma(v) > 0$ is a mortality of young trees which is allowed to depend on the old-tree density v, $f > 0$ is an aging rate, and $h > 0$ is a mortality of old trees. For w, the Dirichlet boundary conditions are imposed on the boundary $\partial\Omega$.

The initial value (u_0, v_0, w_0) is taken from the space

$$K = \{(u_0, v_0, w_0); 0 \le u_0,\ v_0 \in L^\infty(\Omega) \text{ and } 0 \le w_0 \in L^2(\Omega)\}.$$

And Problem (FK) is handled in the underlying product space $X = L^\infty(\Omega) \times L^\infty(\Omega) \times L^2(\Omega)$.

The domain Ω is a $\mathcal{C}^2$ or convex, bounded domain in $\mathbb{R}^2$. We assume as in [4] that the mortality of young trees is given by a square function of the form

$$\gamma(v) = a(v-b)^2 + c, \tag{1}$$

where a, b, $c > 0$ are positive constants. This means that the mortality takes its minimum when the old-age tree density is a specific value b. As mentioned, d, f, h, α, $\beta > 0$ are all positive constants and $0 < \delta \le 1$.

In this talk, we intend to construct a global solution to (FK) for each initial function $U_0 \in K$ and to construct a dynamical system $(S(t), K, X)$ determined from the problem. Furthermore, we notice that $(S(t), K, X)$ enjoys a Lyapunov function and, using the

Lyapunov function, we investigate the omega limit set $\omega(U_0)$ for $U_0 \in K$. Especially, we will show a remarkable fact that, even if U_0 is an intial value consisting of continuous functions, $\omega(U_0)$ can contain an element $(\overline{u}, \overline{v}, \overline{w})$ which consists of some discontinuous functions.

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