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# Direct, Inverse and Control Problems for PDE's - DICOP

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## BOOK OF ABSTRACTS

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## Stabilization properties of memory feedbacks for general decaying kernels

Fatiha ALABAU - BOUSSOUIRA

*INRIA Projet CORIDA, Université Paul Verlaine-Metz, LMAM, CNRS*

We study stabilization properties of visco-elastic materials. For such materials, the feedback operator acts as a convolution operator in time of space second order derivatives of the solution with a time-dependent kernel. We present some of the recent advances for very general decaying kernels.

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## Sharp convergence rate of Glimm scheme for general nonlinear hyperbolic systems

Fabio ANCONA

*Università di Bologna*

Consider the Cauchy problem for an $N$-dimensional, strictly hyperbolic, quasilinear system

$$u_t + A(u)u_x = 0, \qquad u(0,x) = \bar{u}(x), \tag{1}$$

where $u \mapsto A(u)$ is a smooth matrix-valued map, and the initial data $\overline{u}$ is assumed to have small total variation. We investigate the rate of convergence of approximate solutions of (1) constructed by the Glimm scheme.

Relying on an adapted wave tracing method, we will show how to obtain the same tipe of error estimates valid for Glimm approximate solutions of hyperbolic systems of conservation laws $u_t + F(u)_x = 0$ satisfying the classical Lax or Liu assumptions on the eigenvalues $\lambda_k(u)$ and on the eigenvectors $r_k(u)$ of the Jacobian matrix $A(u) = DF(u)$. Namely, letting $u(t,\cdot)$ be the (unique) vanishing viscosity solution of (1) with initial data $\overline{u}$, the following a-priori bound holds

$$\big\|u^\varepsilon(T,\cdot) - u(T,\cdot)\big\| = o(1)\cdot\sqrt{\varepsilon}\,|\log\varepsilon| \tag{2}$$

for an approximate solution $u^\varepsilon$ of (1) constructed by the Glimm scheme, with mesh size $\Delta x = \Delta t = \varepsilon$, and with a suitable choice of the sampling sequence.

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## Vibration testing for detection of small pertubations of an interface

Elena BERETTA

*Università "La Sapienza" Roma*

We present some results concerning the detection of a small perturbation of the interface of a conductivity inclusion from measurements of eigenvectors and eigenvalues associated with the transmission problem for the Laplacian. We derive an asymptotic formula for the perturbations in the modal measurements that are due to small changes in the interface of the inclusion. Using fine gradient estimates, we carefully estimates the error term in this asymptotic formula. We then provide a key dual identity which naturally yields to the formulation of an optimization algorithm for the reconstruction of the interface perturbation. The viability of our reconstruction approach is documented by a variety of numerical results.

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## Optimal control and regularity of boundary traces for some interactive PDE systems

Francesca BUCCI

*Università degli Studi di Firenze*

The talk will focus on established PDE models describing acoustic-structure or fluid-structure interactions. The solvability of the associated quadratic optimal control problem will be discussed, when the system is subject to (boundary) control on the interface. This issue calls for a careful PDE analysis of a related uncontrolled system, and specifically the proof of (sharp) regularity results for the traces of one component of the corresponding solution.

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## About the boundary controllability of coupled parabolic equations

Luz DE TERESA

*Universidad Nacional Autónoma de México*

In this talk we will analyze the null and approximate controllability of the following system:

$$\begin{cases} y_t - \nu\Delta y = 0 & \text{in } \; Q = (0,T)\times\Omega\,, \\ y(t,x) = v & \text{on } (0,T)\times\Gamma_0\,, \\ y(t,x) = 0 & \text{on } (0,T)\times\partial\Omega\backslash\Gamma_0 \\ y(.,0) = y^0 & \text{in } \; \Omega\,, \end{cases}$$

$$\begin{cases} q_t - \Delta q = y & \text{in } \; Q\,, \\ q(t,x) = 0 & \text{on } (0,T)\times\partial\Omega\,, \\ q(.,0) = q^0 & \text{in } \; \Omega\,. \end{cases}$$

where $\Gamma_0 \subset \partial\Omega$ is a non empty open subset of the boundary.

We will show that, in contrast with the distributed case, the controllability results depend on $\nu$. We will give a characterization of $\nu$ in order to get the approximate controllability of the system. When $\nu = 1$ we show the null controllability of the system using Fattorini- Russell technique. Joint work with Enrique FERNÁNDEZ-CARA

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## An ultraparabolic problem arising from age–dependent population diffusion

GABRIELLA DI BLASIO

*Università di Roma "La Sapienza"*

We study well–posedness and regularity results for solutions to a class of differential equations of the form

$$u_t(t,a,x) + u_a(t,a,x) = -\mu(t,a,x)u(t,a,x) + \Delta u(t,a,x), \quad t > 0,\ a > 0,\ x \in \Omega,$$

supplemented by the boundary conditions

$$u(t,0,x) = \int_0^{+\infty} \beta(t,\alpha,x)u(t,\alpha,x)\,d\alpha,\ t > 0,\ x \in \Omega;\ u(t,a,x) = 0, t > 0, a > 0, x \in \partial\Omega,$$

and initial condition

$$u(0,a,x) = u_0(a,x),\ a > 0,\ x \in \Omega.$$

Problem of this kind occur in the study of the dynamics of a population subject to birth, death and diffusion, in a given domain $\Omega \subset \mathbb{R}^n$, where the function $u(t,a,x)$ represents the density of members of age $a$ at time $t$ and position $x$. The terms $d(t,a,x)$ and $b(t,x)$ defined by

$$d(t,a,x) := \mu(t,a,x)u(t,a,x), \quad b(t,x) := \int_0^{+\infty} \beta(t,\alpha,x)u(t,\alpha,x)\,d\alpha$$

represent the death process and the birth process, and the functions $\mu$ and $\beta$ denote the mortality rate and the fertility rate, respectively.

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## Minimal state in viscoelasticity and applications to PDEs

Mauro FABRIZIO

*Università di Bologna*

We show the impact on the initial-boundary value problem of the use of a new notion of state based on the stress-response. Comparisons are made between this new approach and the traditional one, which is based on the identification of histories as states. We shall refer to a stress-response definition of state as the minimal state. Material with memory and with elastic relaxation are discussed. Finally, we show how the evolution of a linear viscoelastic system can be described through a strongly continuous semigroup of linear contraction operators on a appropriate Hilbert space. The family of all solutions of the evolutionary system, obtained by varying the initial data in such a space, is shown to have exponentially decaying energy.

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## Singular limits of the Navier-Stokes-Fourier system on large domains

Eduard FEIREISL

*Institute of Mathematics of the Academy of Sciences of the Czech Republic*

We discuss the problem of singular limits for various dimensionless parameters tending to zero in the full Navier-Stokes-Fourier system posed on "large" spatial domains. Here large means that the boundary of the domain cannot be reached by the acoustic waves in a given time interval. The effect of dispersion on compactness of the velocity field is discussed together with the asymptotic limit.

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## Analysis, optimal control and controllability of a model for the solidification of an alloy

Enrique FERNANDEZ-CARA

*University of Sevilla, Spain*

Some results concerning the theoretical analysis, the optimal control and the controllability of a parabolic system that can be used to model solidification processes will be presented.

It will be assumed that two different kinds of crystallization are possible. Accordingly, the unknowns will be the temperature $\tau$, two phase field functions $u$ and $v$ corresponding to solid fractions and a third phase filed function $w$ associated to the liquid fraction. The time derivatives $u_t$ and $v_t$ appear in the equation for $\tau$ (the heat equation). On the other hand, the equations for $u$ and $v$ contain nonlinear terms where we find $\tau$.

The results have been obtained in collaboration with J. L. Boldrini (Univ. of Campinas, Brazil) and B. Caretta, (Univ. of Campina Grande, Brazil).

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## Motion of solids involving collisions


Michel FRÉMOND

*Università di Roma Tor Vergata*


Let consider a deformable solid evolving above a plane. During its motion it may collide the plane. We investigate the collision phenomenon, i.e., the collision time is known and the unknown is the velocity field after collision. We prove that within reasonable mechanical assumptions this problem has one and only one collision.

Then we investigate the motion of the solid, i.e., the collisions time are not known. There may be many and they may accumulate. We prove, again that within reasonable mechanical assumptions, that there is a motion which has bounded variation with respect to time and smooth behaviour with respect to space. Moreover we prove that the cinetic energy is a bounded variation function in as required by mechanics. When the collision times accumulate, we prove that the motion which follows the accumulation time is smooth and has a zero initial velocity.

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## Long term behavior of binary fluid mixture flows


Ciprian GAL

*University of Missouri, USA*


The quenching of a system from a disordered phase into an ordered one produces a time dependent growth process of ordered regions. The evolution of these regions is the subject of phase ordering dynamics. In the late 1950's, Cahn and Hilliard, were among the first to address these questions. In particular, they investigated behavior of binary alloys. Similar phenomena occur in the phase separation of binary fluids, that is, fluids composed by either two phases of the same chemical species, or phases of different composition. In this case, however, the phenomenology is much more complicated because of the interplay between the phase separation stage and the fluid dynamics. In this talk, a model for the study of incompressible two-phase fluid flows, such as the mixing of two fluids in a driven cavity, or the spinodal decomposition under shear is considered.

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## On the Extensible Thermoelastic Beam


Claudio GIORGI

*Università di Brescia*


This talk is focused on the dissipative system

$$\begin{cases} \partial_{tt}u + \partial_{xxxx}u + \partial_{xx}\theta - \left(\beta + \|\partial_x u\|^2_{L^2(0,1)}\right)\partial_{xx}u = f \\ \partial_t\theta - \partial_{xx}\theta - \partial_{xxt}u = g \end{cases} \tag{TEB}$$

describing the mechanical and thermal evolution of an extensible thermoelastic beam, where the dissipation is entirely contributed by the second equation, ruling the evolution of the absolute temperature $\theta$. This model equation amounts to a 'mild' quasi-linear version for the small transversal deflection of the Euler-Bernoulli thermoelastic beam. As pointed out by [9], it occurs when the geometric nonlinearity, which accounts for the axial tension due to elongation, is taken into consideration.

Under natural boundary conditions, we prove the existence of bounded absorbing sets for translation-bounded functions $f$ and $g$. The absorbing set, besides giving a first rough estimate of the dissipativity of the system, is the preliminary step to prove the existence of much more interesting objects describing the asymptotic dynamics, such as global or exponential attractors [1,2,6,7,8]. Unfortunately, in certain situations where the dissipation is very weak, a direct proof of the existence of the absorbing set via *explicit* energy estimates might be very hard to find (see, for instance, [3,5]). This is the case of problem (TEB), which is also weakly dissipative because the dissipation of the system is contributed by the thermal component only. In particular, due to presence of coupling and nonlinear terms, when performing the standard (and unavoidable) estimates, some "pure" energy terms having a power strictly greater than one pop up with the wrong sign, which are impossible to handle by means of standard Gronwall-type lemmas. Nonetheless, we are still able to establish the result, leaning on some novel Gronwall-type lemma with parameter devised in [4].

But if we assume $f$ and $g$ independent of time, there is a way to define a Lyapunov functional (actually, for an equivalent problem), which would allow to exploit the method depicted in [3,5,6]. In which case, the related semigroup of solutions is shown to possess the global attractor of optimal regularity for all parameters $\beta \in \mathbb{R}$ and the absorbing set is then recovered as a byproduct. Of course, for values of $\beta$ above the buckling limit (which is the same as in the purely mechanical case) we obtain the exponential stability of the (unique) null solution. A joint work with: Vittorino Pata (Poli-Milano) and Maria Grazia Naso (Brescia)

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## Controllability of transport equations in dispersive limit

Olivier GLASS

*Université Pierre et Marie Curie - Paris 6*

We are interested in a one-dimensional transport equation in a bounded interval, perturbed by a small third order dispersive term. This gives a linearized Korteweg-de Vries equation. We prove that if the time of control is long enough, then the cost of the null controllability decreases exponentially as the dispersive term vanishes. We also prove that if on the contrary this time is small enough, then the cost increases exponentially. We also consider the case of a transport equation perturbed by small dissipative and dispersive terms. This is a joint work with Sergio Guerrero.

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## Recent Developments in Wentzell Boundary Conditions

Gisèle Ruiz GOLDSTEIN

*University of Memphis*

Of concern is the damped wave equation

$$u_{tt} + \alpha u_t + Au = 0 \tag{1}$$

where $\alpha > 0$ and $A$ is a uniformly elliptic formally symmetric operator defined for functions on a smooth bounded domain $\Omega$ in $R^n$. We consider dynamic boundary conditions of the form

$$u_{tt} + \beta(x)\partial_n^A u + \alpha u_t + \gamma(x)u = 0 \tag{2}$$

where $\beta$, $\gamma$ are continuous functions on $\partial\Omega$, $\partial_n^A$ is the conormal derivative of $u$ with respect to the operator $A$, $\gamma(x) \geqslant 0$, $\gamma$ is not identically zero, and $\beta(x) > 0$. This operator $A$ is symmetric and has compact resolvent on a suitable $L^2$ space. Overdamping can occur, and the optimal exponential decay rate is explicitly computable as a function of $\alpha$ and $\lambda_1$, the principal or minimal eigenvalue of $A$. These results are joint with J. Goldstein and G. Perla M.

One can also view the equation (1) as the telegraph equation. In the case of certain unbounded domains, where $A$ may or may not have eigenvalues, the solution $u$ of (1) asymptotically equals the solution of the heat equation

$$v(t) = \exp\left(-\frac{tA}{\alpha}\right) h$$

in the sense that

$$\|u(t) - v(t)\| = \|v(t)\|o(1)$$

as $t \to +\infty$. Here $h = F(u(0), u_t(0), \alpha)$ and $F$ can be explicitly computed. The latter results are joint work with T. Clarke, J. Goldstein and S. Romanelli.

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## Phase transition systems with dynamic boundary conditions

Maurizio GRASSELLI

*Politecnico di Milano*

This talk is devoted to the mathematical analysis of phase-field systems of Caginalp type subject to dynamic boundary conditions. We recall that such systems model phase transitions in two-phase materials through the (relative) temperature $\vartheta$ and the order parameter (or phase-field) $\chi$. We present some recent results for models where the coupling between $\vartheta$ and $\chi$ is also on the boundary or is nonlinear (i.e., quadratic) in the bulk. In particular, we intend to discuss the following issues: well-posedness, global asymptotic behavior (i.e., attractors) and convergence to single equilibria.

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## Exact Controllability of the N-dimensional Navier-Stokes equations with N-1 scalar controls

Sergio GUERRERO RODRIGUEZ

*Université Pierre et Marie Curie – Paris 6*

In this talk, we deal with the incompressible Navier-Stokes equations in dimension 2 or 3 with homogeneous Dirichlet boundary conditions on a bounded and regular domaine $\Omega$. We consider the associated control problem where the control function acts through the right-hand side over a small open set $\omega \subset\subset \Omega$. For this system, the *local exact controllability to the trajectories* with $N$ scalar controls is very well-known (see, for instance, [1] or [2]).

In the more recent paper [3], the authors used the results in [2] to establish the local exact controllability to the trajectories of the same system with $N-1$ scalar controls provided that the control domain $\omega$ 'touches' the boundary $\partial\Omega$. The objective of this talk is to prove that this last condition is not necessary.

The idea of the proof is as follows: first, we establish the null controllability of a linearized system associated to the Navier-Stokes equations with $N-1$ scalar controls. Then, we use a fixed-point argument.

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## Partial reconstruction of a source term in a linear parabolic problem

Davide GUIDETTI

*Università di Bologna*

We consider an abstract inverse problem of the form

$$\begin{cases} D_t u(t,x,y) = A(t,x,D_x)u(t,x,y) + B(t,y,D_y)u(t,x,y) + g(t,x)f(t,x,y), \\ (t,x,y) \in [0,T] \times \mathbb{R}^m \times \mathbb{R}^n, \\ u(0,x,y) = u_0(x,y), \quad (x,y) \in \mathbb{R}^m \times \mathbb{R}^n, \\ u(t,x,0) = \phi(t,x), \quad (t,x) \in [0,T] \times \mathbb{R}^m, \end{cases} \tag{1}$$

with $u$ and $g$ unknown. $A$ and $B$ are strongly elliptic operators of order $2p$, in $\mathbb{R}^m$ and $\mathbb{R}^n$ respectively. The last equation in (1) provides the further information, which is necessary to identify $u$ together with $g$. Under suitable assumptions, we are able to prove a result of existence and uniqueness of a global solution.

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## Strong $L^p$-solutions to certain fluid-solid interaction problems

Matthias HIEBER

*University of Darmstadt*

In this talk we consider the the movement of a rigid body in a Newtonian fluid under the influence of gravitation. We show that the system consisting of the Navier-Stokes equations coupled with the balance laws for the momentum and the angular momentum admits a unique, local, strong solution in the $L^p$-setting. Note that the fluid-solid interface is a moving one and has to be found as part of the solution process. This is joint work with Karoline Goetze.

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## On a mathematical model for high-speed milling including the dynamics of machine and work-piece

Dietmar HOEMBERG

*Weierstrass Institute for Applied Analysis and Stochastics, Berlin*

In my talk I will discuss a mathematical model that characterizes the interaction between machine, work-piece, and process dynamics for a complex milling system. While the machine dynamics is modelled in terms of a standard multi-body system, the work-piece is described as a linear thermo-elastic continuum. The coupling of both parts is realized by an empirical process model permitting an estimate of heat and coupling forces occurring during milling.

I will briefly describe the governing equations emphasizing the coupling, then a first analytical result will be outlined concerning the well-posedness of the system. I will conclude with some numerical results showing the dynamics of this complex thermo-mechanical system.

(Joint work with Krzysztof Chelminski (Warsaw University of Technology) and Oliver Rott, (WIAS))

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## Increased stability in Cauchy problem for some partial differential equation

Victor ISAKOV

*Wichita State University*

We show that for some elliptic and parabolic differential equations under some natural assumptions conditional stability of the continuation is improving when some lower order coefficients are growing, i.g. when drift term is properly directed cand getting large. In proofs we use Fouries analysis and Carleman type estimates. We discuss related improved stability in some inverse problems including recovery of potential in a stationary Schrodinger equation at higher frequency. Better stability is crucial for applications (in particular for numerical solution) in control theory and inverse problems.

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## An inverse problem for the heat equation in a degenerate free boundary domain

Mykola IVANCHOV

*Ivan Franko National University of Lviv*

In the domain $\Omega_T \equiv \{(x,t) : 0 < x < t^\gamma h(t), 0 < t < T\}$, where $h = h(t) > 0, t \in [0,T]$, is an unknown function, we consider the inverse problem for finding the coefficient $a(t) > 0$, $t \in [0,T]$ in the heat equation

$$u_t = a(t)u_{xx} + f(x,t) \tag{1}$$

with boundary and overdetermination conditions

$$u(0,t) = \mu_1(t), \quad u(t^\gamma h(t), t) = \mu_2(t), \quad t \in [0,T], \tag{2}$$

$$a(t)u_x(0,t) = \mu_3(t), \quad \int\limits_0^{t^\gamma h(t)} u(x,t)dx = \mu_4(t), \quad t \in [0,T]. \tag{3}$$

By the change $y = \dfrac{x}{h(t)}, \sigma = t^\gamma$ we reduce the problem (1)-(4) to the inverse problem for the degenerate parabolic equation

$$v_\sigma = \frac{b(\sigma)\sigma^{\frac{1-\gamma}{\gamma}}}{\gamma g^2(\sigma)} v_{yy} + \frac{yg'(\sigma)}{g(\sigma)} v_y + F(y,\sigma) \tag{4}$$

in the domain $Q_T \equiv \{(x,t) : 0 < y < \sigma, 0 < \sigma < T_1\}$ with the following conditions

$$v(0,\sigma) = \nu_1(\sigma), \quad v(\sigma,\sigma) = \nu_1(\sigma), \quad 0 < \sigma < T_1, \tag{5}$$

$$b(\sigma)v_y(0,\sigma) = g(\sigma)\nu_3(\sigma), \quad g(\sigma)\int\limits_0^{\sigma} v(y,\sigma)d\sigma = \nu_4(\sigma), \quad 0 < \sigma < T_1, \tag{6}$$

where $v(y,\sigma) = u(yh(\sigma^{1/\gamma}), \sigma^{1/\gamma}), b(\sigma) = a(\sigma^{1/\gamma}), g(\sigma) = h(\sigma^{1/\gamma}), \nu_i(\sigma) = \mu_i(\sigma^{1/\gamma}), i \in \{1,2,3,4\}, T_1 = T^\gamma, F(y,\sigma) = \frac{1}{\gamma} f(yh(\sigma^{1/\gamma}), \sigma^{1/\gamma})$.

Under some assumptions we establish existence and uniqueness of solution

$$(a(t), h(t), u(x,t))$$

for the problem (1)-(6) which belongs to the space $C[0,T] \times C^1[0,T] \times C^{2,1}(Q_T) \cap C(\overline{Q}_T)$ such that $a(t) > 0, h(t) > 0$ when $t \in [0,T]$.

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## A positivity principle for parabolic integro-differential equations and final overdetermination


Jaan JANNO

*Tallinn University of Technology*


We prove a positivity principle for solutions of parabolic integrodifferential equations governing heat flow with memory. The proof is essentially based on the usage of the resolvent $k$ of the memory kernel $m$. Namely, we assume $k \geq 0$ and $k' \leq 0$. Other assumptions are in case $k \neq 0$ weaker than in case $k = 0$. For instance, for the source term $\chi$ of the parabolic integro-differential equation it is sufficient to assume $\chi + k * \chi \geq 0$, where $*$ is the time convolution. This assumption is weaker than the assumption $\chi \geq 0$ that occurs in the case of the usual parabolic differential equation. Physically, this is explained by the inertia of the medium. The same remark holds for the second kind boundary condition, too.

Making use of the positivity principle we study some inverse problems with final overdetermination of the temperature. Namely, we prove existence, uniqueness and stability theorems for a linear inverse source problem and two nonlinear inverse coefficient problems for parabolic integro-differential equations. Existence and stability results for coefficient problems are local.

Our method works only in case the memory kernel $m$ is independent of spatial variables. Otherwise it falls into a divergence-nabla type operator and time resolvent is not applicable.

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## Some uniqueness results for parameter identification in nonlinear hyperbolic PDEs

Barbara KALTENBACHER

*University of Stuttgart*

The subject of this talk is motivated by applications in piezoelectric material characterization, which leads to the problem of identifying coefficient functions in nonlinear hyperbolic PDEs. By means of the model problem of determining $c$ in

$$u_{tt} - (c(u_x)u_x)_x = f \text{ in } \Omega = (0,1)$$

from overdetermined boundary measurements

$$c(u_x)u_x = g\,, \quad u = m \quad \text{ on } \partial\Omega = \{0,1\}$$

of $u$, in this talk we will discuss several approaches for showing uniqueness and stability.

The first one is to adapt the idea of integration along characteristics of the hyperbolic PDE [1] and leads to a stability conjecture of the form

$$\|c - \widetilde{c}\|_{L^2} \leqslant C\|m - \widetilde{m}\|_{H^1(0,T)}\,,$$

under appropriate assumptions, cf. [2].

The second approach works with a neighboring identification problem for which a sufficient condition for identifiability can directly be formulated and checked in terms of the given initial and boundary data. Assuming sufficient regularity of the PDE solution, can prove stability in the form

$$\|c - \widetilde{c}\|_{L^\infty} \leqslant C\|g - \widetilde{g}\|_{L^\infty(0,T)}\,,$$

and Lipschitz stable dependence also on the initial and source data in appropriate norms. An analogous result can be shown to hold true also in three space dimensions for coefficient functions of arbitrary finite dimension, cf. [3].

A third possibility arises when excitations at different intensities can be carried out and corresponding measurements can be taken. In that case and with a polynomial coefficient function $c$, an asymptotic expansion of $u$ in therm of the excitation intensity enables a uniqueness result, cf. [4].

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## Uniqueness in nonlinearly coupled PDE systems


Pavel KREJČÍ

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and

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Consider in $\Omega \times (0,T)$, $\Omega \subset \mathbb{R}^N$, the system

$$\theta_t - \Delta\theta = r(\theta, c) \tag{1}$$

$$c_t - \operatorname{div}\left(D(\theta, c)\nabla c\right) = 0 \tag{2}$$

of heat balance and carbon diffusion equations, complemented with nonhomogeneous boundary conditions on $\partial\Omega$

$$\frac{\partial\theta}{\partial\nu} + h(x)(\theta - \theta_\Gamma(x,t)) = 0 \tag{3}$$

$$-D(\theta, c)\frac{\partial c}{\partial\nu} = b(x,t), \tag{4}$$

and initial conditions for $\theta$ and $c$, as a model problem arising in steel hardening processes. Here, $\theta$ represents the absolute temperature, $c$ is the carbon concentration, and $r(\theta, c)$, $D(\theta, c)$, $h(x)$, $\theta_\Gamma(x,t)$, and $b(x,t)$ are given functions. We mainly focus on the proof of uniqueness and Lipschitz continuous data dependence, which is based on a variant of the "$L^p$-Gronwall" inequality (joint work with Lucia Panizzi).

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## Discrete Carleman estimates for elliptic operators and uniform controllability of semi-discretized parabolic equations

Jérome LE ROUSSEAU

*Université de Provence / INRIA*

In 1995 G. Lebeau and L. Robbiano proved a spectral inequality that led to the first proof of the null controllability of parabolic linear equations. Let $\phi_j$ and $\mu_j$, $j \in \mathbf{N}^*$, denote an orthonormal basis of eigenfunctions and the associated eigenvalues for the elliptic operator $A = -\nabla \cdot (\gamma(x)\nabla)$, with $\gamma$ smooth, on an open set $\Omega \subset \mathbf{R}^n$ with homogeneous Dirichlet boundary conditions. The spectral inequality reads

$$\sum_{\mu_j \leqslant \mu} |a_j|^2 \leqslant C e^{C\sqrt{\mu}} \int_\omega |\sum_{\mu_j \leqslant \mu} a_j \phi_j(x)|^2 \, dx, \qquad \forall (a_j)_{j \in \mathbf{N}^*} \subset \mathbf{C}, \tag{1}$$

where $\omega$ is any non-empty open subset of $\Omega$. It thus measures the loss of orthogonality of the eigenfunctions when restricted to $\omega$. This inequality can be obtained through the derivation of a global Carleman estimate for the elliptic operator $-\partial_{x_0}^2 + A$, where $x_0$ is an additional variable.

Here, we shall consider a discrete version $A_h$ of $A$, using finite differences or finite volumes, for quasi-uniform meshes. We derive a global Carleman estimate for the semi-discrete operator $-\partial_{x_0}^2 + A_h$, in which the size of the Carleman large parameter is connected to the discretization step $h$. We can then prove an inequality of the form of (1) for the lower portion of the spectrum of $A_h$, $\mu \leqslant C/h^2$. This in turn leads to the null controllability of that part of the spectrum in the associated parabolic problem. The $L^2$ norm of the remaining of the spectrum can also be estimated by $e^{-C/h}\|u_0\|_{L^2}$, where $u_0$ is the initial condition used in the parabolic control problem. This result can be considered almost-sharp in view of the existing counterexamples that involve the higher end of the discrete spectrum.

This is joint work with Franck Boyer and Florence Hubert (Universities of Marseille). This research was partially supported by l'Agence Nationale de la Recherche under grant ANR JC07_183284. Laboratoire d'Analyse Topologie Probabilités (LATP), CNRS UMR 6632, Universités d'Aix-Marseille, 39 rue F. Joliot-Curie, 13453 Marseille cedex 13, France. On a research leave at Laboratoire POEMS, INRIA Paris-Rocquencourt/ENSTA, CNRS UMR 2706, France.

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## Observability in continuous and discrete time

Paola LORETI

*Università di Roma "La Sapienza"*

In this talk we consider coupled systems as models of spherical shells. We analyze the problem of observability in continuous and discrete time. Also, we analyze the limit problem as the thickness of the shell goes to zero.

# Sturm-Liouville Problems for an Abstract Differential Equation of Elliptic Type in UMD Spaces

Stéphane MAINGOT

*Laboratoire de Mathématiques, UFR ST, Université du Havre*

In this talk we give some new results on Sturm-Liouville abstract problems for second order differential equations of elliptic type in UMD spaces. Existence, uniqueness and maximal regularity of the strict solution are proved using the celebrated Dore-Venni Theorem.

More precisely, we consider the second order abstract differential equation in $X$

$$u''(x) + Au(x) - \omega u(x) = f(x), \quad \text{a.e.} \quad x \in (0,1), \tag{1}$$

together with the abstract boundary conditions of Sturm-Liouville type

$$u'(0) - Hu(0) = d_0, \; u(1) = u_1. \tag{2}$$

where $f \in L^p(0,1;X), 1 < p < \infty$ ($X$ being a complex $X$ Banach space), $d_0, u_1 \in X$, $A, H$ are closed linear operators in $X$ and $\omega$ is some large positive number.

The particularity lies in the boundary condition containing operator $H$, which can be unbounded. This difficulty will require a precise study of a sum of two linear operators. To this end, we assume that:

1. $X$ is a UMD space.
2. $[0,+\infty[ \subset \rho(A_{\omega_0}), \mathbb{R}_- \subset \rho(H)$ and
$$\sup_{\lambda \geqslant 0} \left\| \lambda \left( A_{\omega_0} - \lambda I \right)^{-1} \right\|_{L(X)} < +\infty \text{ and } \sup_{\lambda \geqslant 0} \left\| \lambda (H + \lambda I)^{-1} \right\|_{L(X)} < +\infty,$$
(here $\omega_0$ is some fixed positive number and $A_{\omega_0} = A - \omega_0 I$).
3. $\forall \; \lambda \geqslant \omega_0, \zeta \geqslant 0, \; (A - \lambda I)^{-1}(H + \zeta I)^{-1} = (H + \zeta I)^{-1}(A - \lambda I)^{-1}$.
4. $\exists C \geqslant 1, \exists \theta_A, \theta_H \in ]0,\pi[$ (with $\dfrac{\theta_A}{2} + \theta_H \in ]0,\pi[$) :
$$\begin{cases} \forall s \in \mathbb{R}, (H)^{is} \in L(X), (-A_{\omega_0})^{is} \in L(X) \text{ and} \\ \|(-A_{\omega_0})^{is}\|_{L(X)} \leqslant C e^{\theta_A |s|}, \left\| (H)^{is} \right\|_{L(X)} \leqslant C e^{\theta_H |s|}. \end{cases}$$

Then, our main result is the following:

**Theorem** *Under the previous assumptions and for $\omega > 0$ large enough, the following assertions are equivalent*

1. $d_0 \in \left( D\left( A \right), X \right)_{\frac{1}{2p} + \frac{1}{2}, p}, \; u_1 \in \left( D\left( A \right), X \right)_{\frac{1}{2p}, p}.$

2. *Problem (1) and (2) has a strict solution $u$, that is*

$$u \in W^{2,p}(0,1;X) \cap L^p(0,1;D(A)), u(0) \in D(H)$$

*and $u$ satisfies (1) and (2).*

*Moreover, in this case, $u$ is uniquely determined by an explicit representation formula.*

Joint work with M. Cheggag, A. Favini, R. Labbas, and A. Medeghri.

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## Well posedness of the finite difference scheme for a singular diffusion problem with a free boundary

Gabriela MARINOSCHI

*Institute of Mathematical Statistics and Applied Mathematics, Bucharest*

We study a finite difference scheme for a nonlinear parabolic equation (possibly degenerate) involving a multivalued coefficient. The model describes the diffusion in a porous medium with the formation of a free boundary. We consider singular boundary conditions which may contain the multivalued function as well. We prove the stability and the convergence of the scheme, emphasizing the precise nature of the convergence in this specific case and determine the error level of the approximating solution. The method is aimed to simplify the numerical computations for the solutions to this type of equations without performing an approximation of the multivalued function. The theory is illustrated by numerical results.

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## Null controllability properties of some degenerate parabolic equations

Patrick MARTINEZ

*Université Toulouse 3*

Motivated by several problems, e.g. the Crocco equation in fluid dynamics, the Fleming-Viot problem in biology, the Black-Scholes equation in economics, we are interested in controllability properties of parabolic equations degenerating at the boundary of the space domain. Typical models of such problems are:

- the following one dimensional case

$$u_t - (a(x)u_x)_x = f(x,t)\chi_{(a,b)}(x), \quad x \in (0,1), t > 0, \tag{1}$$

where the control region $(a,b)$ is a subdomain of $(0,1)$ and the function $a : [0,1] \to \mathbb{R}_+$ is continuous, of classe $C^1$ on $(0,1)$, and $a(0) = 0 = a(1)$;

- the $N$-dimensional case:

$$u_t - \text{div } (A(x)\nabla u) = f(x,t)\chi_\omega(x), \quad x \in \Omega, t > 0, \tag{2}$$

where the control region $\omega$ is a subdomain of $\Omega$, and the matrix $A(x)$ is definite positive for all $x \in \Omega$, and but has at least one eigenvalue equal to 0 for all $x \in \partial\Omega$.

This talk is focused on the $N$-dimensional case. Mainly, we assume that

- the least eigenvalue of the matrix $A(x)$ behaves as $d(x, \partial\Omega)^\alpha$, where $d(x, \partial\Omega)$ is the distance between $x$ and the boundary of the domain $\Omega$, and $\alpha \geqslant 0$,

- the degeneracy occurs in the normal direction: the associated eigenvector is the unit outward vector when $x \in \partial\Omega$.

Then three different cases have to be distinguished: (i): $\alpha \in [0, 1)$, (ii): $\alpha \in [1, 2)$, (iii): $\alpha \geqslant 2$.

In cases (i) and (ii), we derive new Carleman estimates for the adjoint degenerate parabolic equation, and the null controllability of the original problem; the proof of the Carleman estimates is based on some suitable Hardy type inequalities, and on the sharp regularity properties of the solution of the degenerate problem, studied in particular in the thesis of Dario Rocchetti (2008).

In case (iii), we prove that the problem is not null controllable, using earlier results of S. Micu and E. Zuazua (2001), L. Escauriaza, G. Seregin, V. Šverák (2004) concerning the null controllablity properties of the nondegenerate parabolic equations in unbounded domains.

These results were obtained in collaboration with P. Cannarsa (Univ Tor Vergata, Roma 2), and J. Vancostenoble (Univ Toulouse 3).

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## An identification problem with evolution on the boundary of hyperbolic type

Francesca MESSINA

*Università degli Studi di Milano*

We consider an equation of the type

$$A(u + k * u) = f, \tag{1}$$

where $A$ is a linear second-order elliptic. operator, $k$ is a scalar function depending on time only and $k * u$ denotes the standard time convolution of functions defined in $(-\infty, T)$ with their supports in $[0, T]$. The previous equation is endowed with a second-order dynamical boundary condition of the flux form,

$$D_t^2 u + bD_{\nu_A} u = g, \qquad \text{on} \quad (0, T) \times \partial\Omega,$$

where $D_{\nu_A}$ stands for the conormal derivative.
The identification problem we deal with in the present paper is concerned with identifying the unknown convolution *scalar* kernel $k$. To recover $k$ we prescribe an additional condition of the form

$$\Phi[u(t, \cdot)] = l(t), \quad t \in [0, T],$$

$\Phi$ and $l$ being a given functional and a given function. Global existence, uniqueness and continuous dependence results can be shown.
To our knowledge the problem of determining the memory kernel $k$ is new under dynamical boundary conditions.
First we observe that our treatment to solve our identification problem leads to the following (equivalent) identification problem, for the new unknown $v$ such that

$$v = u + k * u \quad \Longleftrightarrow \quad u = v + h * v,$$

$$Av = f \quad \text{in } (0,T) \times \Omega, \tag{2}$$
$$D_t^2 v + D_t h * D_t v + h(0) D_t v + u_0 D_t h + b D_{\nu_A} v + b h * D_{\nu_A} v = g \quad \text{on } (0,T) \times \partial\Omega, \tag{3}$$
$$v(0,\cdot) = u_0 \quad \text{on } (0,T) \times \partial\Omega, \tag{4}$$
$$D_t v(0,\cdot) = u_1 - h(0) u_0 \quad \text{on } (0,T) \times \partial\Omega, \tag{5}$$
$$\Phi[v(t,\cdot)] + h * \Phi[v(t,\cdot)] = l(t), \quad t \in [0,T]. \tag{6}$$

Note that the unknown kernel $h$ convolves not only with the space operator $D_{\nu_A} v$, but also with the time derivative $D_t v$.
For such a kind of identification problems, to the authors' knowledge, the global in time existence of the solution $(v,h)$ is not a standard result.
In this paper this difficulty is overcome by using a suitable estimate strategy and a fixed-point approach strictly related to the problem itself - we could say "suggested" by the problem itself. In particular we transform our problem into an equivalent problem related to the boundary of $\Omega$.
Finally, we observe that a simple treatment of the elliptic problem (1) (endowed with auxiliary nonhomogeneous Dirichlet conditions) forces to choose for $u$ - and consequently for $v$ - an $L^2(\Omega)$-framework.
In contrast to this, we have chosen for the time dependence of $v$ and $h$ the Hölder spaces $C^\alpha$, $\alpha \in (0,1)$, to ensure a good regularity for the unknown kernel $h$ - and consequently for $k$.

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## The Caginalp phase-field system with singular potentials and dynamic boundary conditions

Alain MIRANVILLE

*Université de Poitiers*

Our aim in this talk is to discuss the well-posedness and the asymptotic behavior of the Caginalp phase-field system with a singular potential and dynamic boundary conditions,

$$\begin{cases} \eta \frac{\partial w}{\partial t} - \Delta w = -\frac{\partial u}{\partial t}, \ \eta > 0, \\ \delta \frac{\partial u}{\partial t} - \Delta u + f(u) = w, \ \delta > 0, \\ \dfrac{\partial w}{\partial \nu}\Big|_\Gamma = 0, \quad u|_\Gamma = \psi, \\ \frac{\partial \psi}{\partial t} - \Delta_\Gamma \psi + \lambda \psi + g(\psi) + \dfrac{\partial u}{\partial \nu} = 0 \text{ on } \Gamma, \ \lambda > 0, \end{cases}$$

in a bounded and smooth domain $\Omega \subset \mathbb{R}^3$ with boundary $\Gamma$. Here, $w$ denotes the temperature and $u$ denotes the order parameter; this system was proposed by G. Caginalp in order to model melting-solidification phenomena in certain classes of materials. Furthermore, $\Delta_\Gamma$ is the Laplace-Beltrami operator and $\nu$ is the unit outer normal to $\Gamma$. Finally, $f$ is singular, say, at $-1$ and 1 (typically, we have in mind the following thermodynamically relevant potential:

$$f(s) = -2\kappa_0 s + \kappa_1 \ln \frac{1+s}{1-s},\ s \in (-1,1),\ 0 < \kappa_0 < \kappa_1)$$

and $g$ is regular.

These dynamic boundary conditions have been proposed by physicists, in the context of the Cahn-Hilliard equation for phase separation (note that, if we take $\eta = \delta = 0$, then we recover the Cahn-Hilliard equation), in order to account for the interactions with the walls in confined systems.

The main difficulty here is to prove proper separation properties (and corresponding dissipative estimates).

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## Stabilization of second order evolution equations with unbounded feedback with delay

Serge NICAISE

*University of Valenciennes*

We consider abstract second order evolution equations with unbounded feedback with delay. Existence results are obtained under some realistic assumptions. Sufficient and explicit conditions are derived that guarantee the exponential or polynomial stability. Some new examples that enter into our abstract framework are presented.

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## Linear evolution equation of hyperbolic type with application to Schrödinger equations

Noboru OKAZAWA

*Department of Mathematics Science University of Tokyo*

Let $\{A(t); 0 \leqslant t \leqslant T\}$ be a family of closed linear operators in a complex Hilbert space $X$. This talk is concerned with linear evolution equations of the form

$$du(t)/dt + A(t)u(t) = f(t) \quad \text{on} \quad (0,T). \tag{E}$$

Let $S$ be a selfadjoint operator in $X$, satisfying $(u, Su) \geqslant \|u\|^2$ for $u \in D(S)$. Assume that the following four conditions are satisfied:

**(I)** There is $\alpha \in L^1(0,T)$, $\alpha \geqslant 0$, such that

$$|\mathrm{Re}\,(A(t)v, v)| \leq \alpha(t)\|v\|^2,\ v \in D(A(t)),\ \text{a.a. } t \in (0,T);$$

**(II)** $Y := D(S^{1/2}) \subset D(A(t)),\ 0 \le t \le T$;

**(III)** There is $\beta \in L^1(0,T)$, $\beta \ge \alpha$, such that

$$|\mathrm{Re}\,(A(t)u, Su)| \le \beta(t)\|S^{1/2}u\|^2,\ u \in D(S),\ \text{a.a. } t \in (0,T);$$

**(IV)** $A(\cdot) \in L^2\big(0,T;B(Y,X)\big)$.

Then we have

**Theorem.** Let $f(\cdot) \in L^2(0,T;X) \cap L^1(0,T;Y)$. Then there exists a unique strong solution $u(\cdot)$ of (E) with $u(0) = u_0 \in Y$ such that $u(\cdot) \in H^1(0,T;X) \cap C([0,T];Y)$.

Now let $k \in \mathbb{N}$. Then, by introducing

$$S := 1 + \Delta^{2k} + |x|^{4k}, \quad D(S^{1/2}) = H^{2k}(\mathbb{R}^N) \cap D(|x|^{2k}),$$

the above-mentioned theorem can be applied to the Cauchy problem for Schrödinger evolution equations ($i := \sqrt{-1}$):

$$i\,du(t)/dt - \big(-\Delta + V(x,t)\big)u(t) = 0, \quad 0 < t < \infty, \tag{SE}$$

in $L^2(\mathbb{R}^N)$. The assumption is satisfied under the following three conditions:

**(V0)** $V(\cdot,t) \in C^{2k}(\mathbb{R}^N)$ a.a. $t \in (0,\infty)$;

**(V1)** $(1 + |x|^{2k})^{-1}V \in L^2_{\mathrm{loc}}([0,\infty); L^\infty(\mathbb{R}^N))$;

**(V2)** $\exists\ g_j \in L^1_{\mathrm{loc}}[0,\infty);\ \sum_{|\alpha|=j} |D^\alpha V(x,t)| \le g_j(t)(1 + |x|^j), \quad 1 \le j \le 2k.$

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## Convergence of solutions of nonlocal phase-separation models to equilibria

Luciano PANDOLFI

*Politecnico di Torino*

Gurtin-Pipkin equation has been introduced to take into account memory effect in heat processes. It is the equation

$$\theta'(t) = \int_0^t N(t-s)\theta(s)\,\mathrm{d}s \tag{\textbf{G-P}}$$

where $\theta = \theta(t,x)$, where $t > 0$ and $x \in \Omega$, a suitable region of $\mathbf{R}^n$.

When supplied with an initial condition $\theta(0) = \xi$ and suitable boundary conditions, this equation identifies a well posed problem.

When a control acts on Equation **G-P**, either distributed or boundary control, we can put a control problem, i.e. we can ask to hit a prescribed target in a given time $T$. This problem has been studied by several authors. The proofs on these papers, based on inverse inequalities and Carleman estimates (see [1,2]) or compactness arguments (see [3]) are not constructive. This fact, and the fact that when $N(t) = 1$ Eq. **(G-P)** is an integrated form of the wave equation, suggest that we consider the special case $\Omega = (0, \pi)$ and we try a constructive approach to the controllability problem in the case the control acts on the boundary,

$$\theta(t, 0) = u(t)\,, \qquad \theta(t, \pi) = 0\,.$$

We shall present several observation on the applicability of the moment problem to the solution of this control problem.


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## Convergence of solutions of nonlocal phase-separation models to equilibria


Hana PETZELTOVA

*Institute of Mathematics of the Academy of Sciences of the Czech Republic*


We will discuss convergence of solutions of phase-transitions systems which are nonlocal in space. These models take into account interactions between states in both short and long scales, which is expressed by a convolution with a suitable kernel. Solutions of the models in question satisfy the energy inequality, however, the corresponding energy functionals are not twice continuously differentiable on the spaces where compactness of trajectories has been proved. This makes impossible an application of the standard Łojasiewicz inequality, which is commonly used in proofs of convergence of solutions of phase-field systems, and requires to employ a non-smooth version of this inequality.

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## On a semilinear elliptic boundary value problem


Cristina PIGNOTTI

*Università degli Studi dell'Aquila*


We prove an existence result of positive solutions of the semilinear elliptic equation

$$\begin{cases} \Delta u & = W'(u), \quad x \in \Omega, \\ u & = 0, \qquad\quad x \in \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^n$ is a Lipschitz domain and $W : \mathbb{R} \to \mathbb{R}$ is a $C^2$ function satisfying suitable assumptions.

Moreover, we give an exponential estimate on the solution and discuss the extension of such an analysis to the case of mixed Dirichlet-Neumann boundary conditions.

Joint work with G. Fusco and F. Leonetti.

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## Data assimilation problems: non standard approach and Tychonov regularisation revisited

Jean-pierre PUEL

*Universite de Versailles*

Data assimilation problems consist in retrieving the value of the solution at some time (everywhere in the domain) for an evolution problem knowing informations on the solution on a subdomain during a period of time. In a classical approach one tries to find the initial data using a Tychonov regularization. This depends on a regularization parameter and the problem becomes ill-posed when this parameter tends to zero. We will present a non standard approach, based on controllability techniques, which enables to retrieve the solution at the final time. A consequence is also to give a sense to the classical approach in a non standard functional class. We will present this on a heat equation or on a linearized Navier-Stokes system.

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## Asymptotics in nonlinear hyperbolic thermoelasticity

Reinhard RACKE

*University of Konstanz*

We consider a hyperbolic model in thermoelasticity and discuss the asymptotic behavior as time tends to infinity of nonlinear models in one space dimension, and we describe the propagation of singularities for linear and semilinear models in three space dimensions. Finally, we look at resolvent expansions in exterior domains.

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## Weighted Hardys Inequality and the Kolmogorov Equation perturbed by an inverse-square potential

Abdelaziz RHANDI

*Università degli Studi di Salerno*

In this talk we give sufficient and necessary conditions for the existence of a weak solution of a Kolmogorov equation perturbed by an inverse-square potential. More precisely, using a weighted Hardy's inequality with respect to an invariant measure $\mu$, we show the

existence of the semigroup solution of the parabolic problem corresponding to a generalized Ornstein-Uhlenbeck operator perturbed by an inverse-square potential in $L^2(\mathbb{R}^N, \mu)$. In the case of the symmetric Ornstein-Uhlenbeck operator we obtain an instantaneous blowup.

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## Symmetric and nonsymmetric operators with general Wentzell boundary conditions

Silvia ROMANELLI

*Università di Bari*

We will deal with some results obtained in the papers [1]-[3] concerning existence, regularity and continuous dependence on the boundary conditions of the $(C_0)$ semigroups generated by the closures of some symmetric and nonsymmetric uniformly elliptic operators with general Wentzell boundary conditions. Here we will focus on the operators acting on the Hilbert space $X_2$ defined as the completion of $C(\overline{\Omega})$ in the norm $|||\cdot|||_2$ given by $|||u|||_2 := \left(\int_\Omega |u|^2\,dx + \int_{\partial\Omega} |u|^2 \frac{dS}{\beta}\right)^{\frac{1}{2}}$, where $\Omega$ is a bounded domain of $\mathbf{R}^N$ with boundary $\partial\Omega$ in $C^{2+\epsilon}$ and $\beta \in C^1(\partial\Omega)$, $\beta > 0$, comes from the boundary conditions.

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## On the Cahn-Hilliard equation with singular potential and dynamic boundary conditions

Giulio SCHIMPERNA

*Università di Pavia*

In this talk we will present some recent results, obtained in collaboration with G. Gilardi and A. Miranville, concerning existence, uniqueness, regularity properties, and long time behavior of solutions to the following equation of Cahn-Hilliard type:

$$u_t - \Delta(-\Delta u + f(u)) = g, \tag{1}$$

where the unknown $u$ lives in a smooth bounded domain $\Omega \subset \mathbb{R}^3$ for times $t \in (0, +\infty)$, $g$ is a volumic source, and no-flux boundary conditions are assumed for the function $w = -\Delta u + f(u)$ (chemical potential).

The main novelty of our problem is represented by the *dynamic* boundary condition assumed for $u$, namely

$$v_t - \Delta_\Gamma v + f_\Gamma(v) = -\partial_{\mathbf{n}} u, \tag{2}$$

where $\Gamma := \partial\Omega$, $v := u|_\Gamma$, $\Delta_\Gamma$ is the Laplace-Beltrami operator on $\Gamma$, and $\partial_{\mathbf{n}}$ denotes the (outer) normal derivative.

The system (1-2), coupled with suitable Cauchy conditions, will be studied in case one or both of the nonlinearities $f$, $f_\Gamma$ are *singular* functions. This means that $f$ and $f_\Gamma$ are increasingly monotone up to a linear perturbation, but they could be defined only inside a bounded interval of the real line (that consisting of the physically admissible values of $u$). A meaningful example is given by the so-called *logarithmic* nonlinearity

$$f(u) = (1-u)\log(1-u) + (1+u)\log(1+u) - \lambda u^2, \qquad \lambda \geq 0,$$

which makes sense for $u \in (-1, 1)$.

We will show that, under rather weak compatibility conditions between $f$ and $f_\Gamma$, a unique weak solution exists. Moreover, we will investigate the long-time properties of the semiflow associated to this problem from the point of view of $\omega$-limits of trajectories as well as of global and exponential attractors.

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## Control problems for integro-differential equations

Daniela SFORZA

*Università di Roma "La Sapienza"*

The aim of this talk is to give a survey of some results in control theory for second order evolution equations with memory. Under suitable assumptions on the convolution kernels, we will show global existence and asymptotic behaviour for the solutions of some nonlinear equations. Moreover, a reachability result for integro-differential equations with exponential type kernels will be given.

All these results can be applied to problems in viscoelasticity.

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## Oscillating thin elastoplastic bodies: dimensional reduction, hysteresis operators, existence results

Jürgen SPREKELS

*Weierstrass Institute for Applied Analysis and Stochastics, Berlin*

We study the dynamics of oscillating thin elastoplastic beams and plates. Using dimensional reduction and the three-dimensional von Mises flow rule of plasticity, we arrive at systems of fourth-order PDEs in which multidemsional hysteresis operators occur. Unique

existence is shown by exploiting special techniques from the theory of hysteresis operators. The results are joint work with P. Krejci (WIAS Berlin) and, in parts, with R.B. Guenther (Oregon State University).

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## Time reversal and data assimilation


MARIUS TUCSNAK

*Nancy Universits, France*


Given a skew-adjoint operator $A : X_1 \longrightarrow X$ generating a $C^0$ group of isometries $\mathbb{T}$ on $X$, consider the system

$$\dot{z}(t) = Az(t), \quad z(0) = x \in X. \tag{1}$$

with the observation

$$y(t) = Cz(t)$$

where $C \in \mathcal{L}(X_1, Y)$ is an admissible observation operator for $\mathbb{T}$. Under the assumption that the pair $(A, C)$ is exactly observable in some time $\tau > 0$ we

Let $\Psi_\tau \in \mathcal{L}(X, L^2([0, \tau]; Y))$ be defined by

$$(\Psi_\tau z_0)(t) = C\mathbb{T}_t z_0 \text{ for } \ t \in [0, \tau], \quad z_0 \in X.$$

We are interested in solving the inverse problem of determining the initial data $x$ from the observation $y$ (the data assimilation problem). In other words, we want to solve the equation

$$\Psi_\tau x = y, \tag{2}$$

where $y \in L^2([0, \tau]; Y)$ is supposed to be a given element of the range of $\Psi_\tau$. This is done by using the version from [1] of the time reversal method of Fink. More precisely we consider the iterative method

$$\begin{cases} \dot{v}^j(t) = (A - C^*C)v^j(t) + 2C^*C(\mathbf{\text{Я}}_\tau e^{j-1})(t), \\ v^j(0) = 0, \end{cases} \qquad \forall j \geq 1 \tag{3}$$

$$e^j(t) = v^j(t) - \mathbf{\text{Я}}_\tau e^{j-1}(t), \tag{4}$$

where $\mathbf{\text{Я}}_\tau \in \mathcal{L}(L^2([0, \tau]; X))$ is the "time reversal operator". The choice of this operator depends on $A$ and $C$. We have, for instance, for a Schrödinger equation with internal observation that

$$(\mathbf{\text{Я}}_\tau v)(t) = \overline{v(\tau - t)} \qquad (v \in L^2(0, \tau; L^2(\Omega)).$$

Our main abstract result asserts that, under the above assumptions, we have

$$\left\| x - \sum_{k=0}^{N} v^{2k}(\tau) \right\| = \|e^{2N}(\tau)\| \leq Me^{-2N\alpha\tau}\|x\|.$$

The above estimate provides an efficient method for reconstructiong initial data. We apply this approach, both theotericaly and numerically, to the wave and to the Schrödinger equations.

Joint work with Karim Ramdani.


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## Hardy's inequalities and singular inverse-square potentials in control theory


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From the works of Baras and Goldstein [1,2], it is well-known that singular inverse-square potentials, arising, for example, in the context of combustion theory and quantum mechanics, generate interesting phenomena: when one replaces the Laplace operator $-\Delta$ by $-\Delta - \lambda|x|^{-2}I$ in the heat equation, (global) existence and non-existence (instantaneous blow-up) of positive solutions is crucially determined by the value of the parameter $\lambda$, the critical value of $\lambda$ being the constant $\lambda^\star := (N-2)^2/4$, that appears in the so-called Hardy inequality.

We present here the results of two joint works with E. Zuazua in which we address the question of null (or exact) controllability of heat, wave and Schrödinger equations perturbed by such inverse-square potentials. Null controllability is proved in the range of sub-critical coefficients $\lambda \leqslant \lambda^\star$ and under suitable geometric conditions. The proofs of the observability inequalities rely on Carleman estimates [4,3] or on the method of multipliers [5], the key point being each time a suitable Hardy-type inequality. On the contrary, in the super-critical case, null controllability $\lambda > \lambda^\star$ becomes false [3,5].


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## An Identification Problem for an Abstract System of Linear Evolution Equations in A Banach Space

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We prove both the existence and uniqueness of a solution to the identification problem: *find an element $z \in X$ and a function $u : [0,1] \to X$ satisfying*

$$(\mathcal{IP}) \quad \begin{cases} u'(t) = Au(t) + F(t)z, & t \in [0,1] \\ u(0) = u_0, & \\ \mu([0,1])^{-1} \displaystyle\int_0^1 d\mu(t)u(t) = u_1, & \end{cases}$$

*where $A$ is the infinitesimal generator of a $C_0$-semigroup in a Banach space $X$, $F \in C^1([0,1]; \mathcal{L}(X))$ and $\mu$ is countably additive $\mathcal{L}(X)$-valued measure on $[0,1]$ with $\mu([0,1])$ being invertible.*

We stress that the present result extends to the case of $C_0$-semigroups the explicit representation formulae for the solution of problem $(\mathcal{IP})$ obtained by Yu. Anikonov and A. Lorenzi [1] in the specific case when $A$ is the infinitesimal generator of an analytic semigroup of contractions and $\mu = \lambda I$, $\lambda$ being a finite Borel measure on $[0,1]$. So, our abstract result handles both parabolic and hyperbolic equations and systems.

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Joint work with Alfredo Lorenzi.

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## Spectral multipliers and Evolution equation

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We study the functional calculus of operators $A$ in an $L_p$-scale which are typically selfadjoint on $L_2$ and sectorial on $L_p$ with spectrum in $\mathbb{R}_+$. Sharp estimates for functions of $A$ are important for representations of the fractional domains of $A$ and regularity estimates for evolution equations governed by $A$.

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# From exact observability to identification of singular sources

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We list some recent results which are relevant in the identification of point sources for the wave equation, using boundary measurements. The results are in a more general framework, dealing with an arbitrary operator semigroup and an admissible observation operator for it. Our standing assumptions are that $\mathbb{T}$ is a strongly continuous semigroup of operators on the Hilbert space $X$, with generator $A : \mathcal{D}(A) \to X$. For a Hilbert space $V$, $m \in \mathbb{N}$ and for $\tau > 0$ we set

$$\mathcal{H}^m_R(0,\tau;V) = \left\{ u \in \mathcal{H}^m(0,\tau;V) \mid u(\tau) = \cdots = u^{m-1}(\tau) = 0 \right\} .$$

We denote $X^d_m = \mathcal{D}((A^*)^m)$, with the graph norm.

**Proposition 1.** *Let $C \in \mathcal{L}(X_1, Y)$ be an admissible observation operator for $\mathbb{T}$. For $\tau > 0$ let $\Psi_\tau$ be the output map corresponding to the pair $(A, C)$. Then, for each $\tau > 0$, $\Psi_\tau$ has a unique continuous extension $\Psi_\tau \in \mathcal{L}((Z^d_m)', [\mathcal{H}^m_R(0,\tau;Y)]')$, where*

$$Z^d_m = X^d_m + (\beta I - A^*)^{-1} C^* U, \tag{0.1}$$

*for some $\beta \in \rho(A)$. Here and below, dualities are computed with respect to the Hilbert spaces $X$ and $L^2([0,\tau];Y)$, respectively.*

*Moreover, assume that $(A, C)$ is exactly observable in some time $\tau_0 > 0$. Then for each $\tau > \tau_0$, there exists a constant $m_\tau > 0$ such that, for every $f \in (Z^d_m)'$, we have*

$$\|\Psi_\tau f\|_{\left[\mathcal{H}^m_R(0,\tau;Y)\right]'} \geqslant m_\tau \|f\|_{(Z^d_m)'} . \tag{0.2}$$

**Theorem 2.** *Let $X$, $Y$ be Hilbert spaces and assume that the pair $(A, C)$ is exactly observable in some time $\tau_0 > 0$ and that $\lambda \in \mathcal{H}^1(0,\tau)$ with $\lambda(0) \neq 0$. We define $\mathbb{F}_\tau f$ as the convolution (on $[0,\tau]$) of $\lambda$ with $\Psi_\tau f$. Then for every $\tau > \tau_0$, $\mathbb{F}_\tau$ is one-to-one from $(Z^d_m)'$ to $[\mathcal{H}^{m-1}_R(0,\tau;Y)]'$ and there exists a positive constant $\tilde{\kappa}_\tau$ such that*

$$\|f\|_{(Z^d_m)'} \leqslant \tilde{\kappa}_\tau \|\mathbb{F}_\tau f\|_{(\mathcal{H}^{m-1}_R(0,\tau;Y))'} \qquad \forall\, f \in (Z^d_m)'. \tag{0.3}$$

The above theorem gives conditions to recover $f$ from $y$, when the system is described by $\dot{z} = Az + \lambda f$, $z(0) = 0$, $y = Cz$ (for $t \in (0,\tau)$). An interesting particular case is when $A$ corresponds to the wave equation and $f$ is the control operator that corresponds to a Dirac mass (point source) acting on the velocity component of the state.

## Irregular Elliptic Problems in UMD Banach Spaces

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We prove coerciveness with a defect and Fredholmness of nonlocal irregular boundary value problems for second order elliptic differential-operator equations in *UMD* Banach spaces. Then, we prove coerciveness with a defect in both the space variable and the spectral parameter of the problem with a linear parameter in the equation. The results do not imply maximal $L_p$-regularity in contrast to previously considered regular case. Finally, application to nonlocal irregular boundary value problems for elliptic equations of the second order in cylindrical domains are presented. Equations and boundary conditions may contain differential-integral parts. The spaces of solvability are Sobolev type spaces $W^{2,2}_{p,q}$.

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## Uniqueness by Dirichlet-to-Neumann map on an arbitrary part of boundary in two dimensions

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We prove for a two dimensional bounded simply connected domain that the Cauchy data for the Schrödinger equation measured on an arbitrary open subset of the boundary determines uniquely the potential. This implies, for the conductivity equation, that if we measure the current fluxes at the boundary on an arbitrary open subset of the boundary produced by voltage potentials supported in the same subset, we can determine uniquely the conductivity. We use Carleman estimates with degenerate weight functions to construct appropriate complex geometrical optics solutions to prove the results. This is a joint work with O. Imanuvilov (Colorado State University) and G. Uhlmann (University of Washington).

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