Title: Sharp convergence rate of Glimm scheme for general nonlinear hyperbolic systems

Abstract: Consider the Cauchy problem for an N-dimensional, strictly hyperbolic, quasilinear system

$$u_t + A(u)u_x = 0, \qquad u(0,x) = \bar{u}(x), \tag{1}$$

where $u \mapsto A(u)$ is a smooth matrix-valued map, and the initial data $\overline{u}$ is assumed to have small total variation. We investigate the rate of convergence of approximate solutions of (1) constructed by the Glimm scheme.

Relying on an adapted wave tracing method, we will show how to obtain the same tipe of error estimates valid for Glimm approximate solutions of hyperbolic systems of conservation laws $u_t + F(u)_x = 0$ satisfying the classical Lax or Liu assumptions on the eigenvalues $\lambda_k(u)$ and on the eigenvectors $r_k(u)$ of the Jacobian matrix $A(u) = DF(u)$. Namely, letting $u(t,\cdot)$ be the (unique) vanishing viscosity solution of (1) with initial data $\overline{u}$, the following a-priori bound holds

$$\big\|u^\varepsilon(T,\cdot) - u(T,\cdot)\big\| = o(1)\cdot\sqrt{\varepsilon}\,|\log\varepsilon| \tag{2}$$

for an approximate solution u^ε of (1) constructed by the Glimm scheme, with mesh size $\Delta x = \Delta t = \varepsilon$, and with a suitable choice of the sampling sequence.