An ultraparabolic problem arising from age–dependent population diffusion

Gabriella Di Blasio

We study well–posedness and regularity results for solutions to a class of differential equations of the form

$$u_t(t,a,x) + u_a(t,a,x) = -\mu(t,a,x)u(t,a,x) + \Delta u(t,a,x), \quad t>0,\ a>0,\ x\in\Omega,$$

supplemented by the boundary conditions

$$u(t,0,x) = \int_0^{+\infty} \beta(t,\alpha,x)u(t,\alpha,x)\,d\alpha,\ t>0,\ x\in\Omega; \quad u(t,a,x)=0,\ t>0,\ a>0,\ x\in\partial\Omega,$$

and initial condition

$$u(0,a,x) = u_0(a,x),\ a>0,\ x\in\Omega.$$

Problem of this kind occur in the study of the dynamics of a population subject to birth, death and diffusion, in a given domain $\Omega\subset\mathbb{R}^n$, where the function $u(t,a,x)$ represents the density of members of age a at time t and position x. The terms $d(t,a,x)$ and $b(t,x)$ defined by

$$d(t,a,x) := \mu(t,a,x)u(t,a,x), \quad b(t,x) := \int_0^{+\infty} \beta(t,\alpha,x)u(t,\alpha,x)\,d\alpha$$

represent the death process and the birth process, and the functions μ and β denote the mortality rate and the fertility rate, respectively.