ON THE EXTENSIBLE THERMOELASTIC BEAM

CLAUDIO GIORGI (BRESCIA)

This talk is focused on the dissipative system

$$(1)\qquad \begin{cases} \partial_{tt}u + \partial_{xxxx}u + \partial_{xx}\theta - \big(\beta + \|\partial_x u\|^2_{L^2(0,1)}\big)\partial_{xx}u = f \\ \partial_t\theta - \partial_{xx}\theta - \partial_{xxt}u = g \end{cases}$$

describing the mechanical and thermal evolution of an extensible thermoelastic beam, where the dissipation is entirely contributed by the second equation, ruling the evolution of the absolute temperature θ. This model equation amounts to a 'mild' quasi-linear version for the small transversal deflection of the Euler-Bernoulli thermoelastic beam. As pointed out by [9], it occurs when the geometric nonlinearity, which accounts for the axial tension due to elongation, is taken into consideration.

Under natural boundary conditions, we prove the existence of bounded absorbing sets for translation-bounded functions f and g. The absorbing set, besides giving a first rough estimate of the dissipativity of the system, is the preliminary step to prove the existence of much more interesting objects describing the asymptotic dynamics, such as global or exponential attractors [1, 2, 6, 7, 8]. Unfortunately, in certain situations where the dissipation is very weak, a direct proof of the existence of the absorbing set via *explicit* energy estimates might be very hard to find (see, for instance, [3, 5]). This is the case of problem (1), which is also weakly dissipative because the dissipation of the system is contributed by the thermal component only. In particular, due to presence of coupling and nonlinear terms, when performing the standard (and unavoidable) estimates, some "pure" energy terms having a power strictly greater than one pop up with the wrong sign, which are impossible to handle by means of standard Gronwall-type lemmas. Nonetheless, we are still able to establish the result, leaning on some novel Gronwall-type lemma with parameter devised in [4].

But if we assume f and g independent of time, there is a way to define a Lyapunov functional (actually, for an equivalent problem), which would allow to exploit the method depicted in [3, 5, 6]. In which case, the related semigroup of solutions is shown to possess the global attractor of optimal regularity for all parameters $\beta \in \mathbb{R}$ and the absorbing set is then recovered as a byproduct. Of course, for values of β above the buckling limit (which is the same as in the purely mechanical case) we obtain the exponential stability of the (unique) null solution.

References

[1] A.V. Babin, M.I. Vishik, *Attractors of evolution equations*, North-Holland, Amsterdam, 1992

[2] V.V. Chepyzhov, M.I. Vishik, *Attractors for equations of mathematical physics*, Amer. Math. Soc., Providence, 2002

[3] M. Conti, V. Pata, *Weakly dissipative semilinear equations of viscoelasticity*, Commun. Pure Appl. Anal. **4** (2005) 705–720

[4] S. Gatti, V. Pata, S. Zelik, *A Gronwall-type lemma with parameter and dissipative estimates for PDEs*, Nonlinear Anal. (to appear).

[5] C. Giorgi, V. Pata, E. Vuk, *On the extensible viscoelastic beam,* Nonlinearity **21** (2008) 713–733

[6] J.K. Hale, *Asymptotic behavior of dissipative systems*, Amer. Math. Soc., Providence, 1988

[7] A. Miranville and S. Zelik, *Attractors for dissipative partial differential equations in bounded and unbounded domains*, in "Handbook of Differential Equations, Evolutionary Partial Differential Equations" (C.M. Dafermos and M. Pokorny, Eds.), Elsevier, Amsterdam, in press

[8] R. Temam, *Infinite-dimensional dynamical systems in mechanics and physics*, Springer, New York, 1997

[9] S. Woinowsky-Krieger, *The effect of an axial force on the vibration of hinged bars*, J. Appl. Mech. **17** (1950) 35–36

A joint work with: Vittorino Pata (Poli-Milano) and Maria Grazia Naso (Brescia)