The Damped Wave Equation with Dynamic Boundary Conditions

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Of concern is the damped wave equation

$$u_{tt} + \alpha u_t + Au = 0 \tag{1}$$

where $\alpha > 0$, and A is a uniformly elliptic formally symmetric operator defined for functions on a smooth bounded domain Ω in $\mathbb{R}^n$. We consider dynamic boundary conditions of the form

$$u_{tt} + \beta(x)\,\partial_n^A u + \alpha u_t + \gamma(x)\,u = 0 \tag{2}$$

where β, γ, are continuous functions on $\partial\Omega$, $\partial_n^A u$ is the conormal derivative of u with respect to the operator A, $\gamma(x) \geq 0$, γ is not identically zero, and $\beta(x) > 0$. This operator A is symmetric and has compact resolvent on a suitable L^2 space. Overdamping can occur, and the optimal exponential decay rate is explicitly computable as a function of α and λ_1, the principal or minimal eigenvalue of A. These results are joint with J. Goldstein and G. Perla M.

One can also view the equation (1) as the telegraph equation. In the case of certain unbounded domains, where A may or may not have eigenvalues, the solution u of (1) asymptotically equals the solution of the heat equation

$$v(t) = \exp\left(-\frac{tA}{\alpha}\right)h$$

in the sense that

$$\|u(t) - v(t)\| = \|v(t)\|\,o(1)$$

as $t \to \infty$. Here $h = F(u(0), u_t(0), \alpha)$ and F can be explicitly computed. The latter results are joint work with T. Clarke, J. Goldstein and S. Romanelli.