AN INVERSE PROBLEM FOR THE HEAT EQUATION IN A DEGENERATE FREE BOUNDARY DOMAIN

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In the domain $\Omega_T \equiv \{(x,t) : 0 < x < t^\gamma h(t), 0 < t < T\}$ where $h = h(t) > 0, t \in [0,T]$ is unknown function we consider the inverse problem for finding the coefficient $a(t) > 0, t \in [0,T]$ in the heat equation

$$u_t = a(t)u_{xx} + f(x,t) \tag{1}$$

with boundary and overdetermination conditions

$$u(0,t) = \mu_1(t), \quad u(t^\gamma h(t),t) = \mu_2(t), \quad t \in [0,T], \tag{2}$$

$$a(t)u_x(0,t) = \mu_3(t), \quad \int_0^{t^\gamma h(t)} u(x,t)dx = \mu_4(t), \quad t \in [0,T]. \tag{3}$$

By the change $y = \dfrac{x}{h(t)}, \sigma = t^\gamma$ we reduce the problem (1)-(4) to the inverse problem for the degenerate parabolic equation

$$v_\sigma = \frac{b(\sigma)\sigma^{\frac{1-\gamma}{\gamma}}}{\gamma g^2(\sigma)} v_{yy} + \frac{yg'(\sigma)}{g(\sigma)} v_y + F(y,\sigma) \tag{4}$$

in the domain $Q_T \equiv \{(x,t) : 0 < y < \sigma, 0 < \sigma < T_1\}$ with the following conditions

$$v(0,\sigma) = \nu_1(\sigma), \quad v(\sigma,\sigma) = \nu_1(\sigma), \quad 0 < \sigma < T_1, \tag{5}$$

$$b(\sigma)v_y(0,\sigma) = g(\sigma)\nu_3(\sigma), \quad g(\sigma)\int_0^\sigma v(y,\sigma)d\sigma = \nu_4(\sigma), \quad 0 < \sigma < T_1, \tag{6}$$

where $v(y,\sigma) = u(yh(\sigma^{1/\gamma}), \sigma^{1/\gamma}), b(\sigma) = a(\sigma^{1/\gamma}), g(\sigma) = h(\sigma^{1/\gamma}), \nu_i(\sigma) = \mu_i(\sigma^{1/\gamma}), i \in \{1,2,3,4\}, T_1 = T^\gamma, F(y,\sigma) = \frac{1}{\gamma} f(yh(\sigma^{1/\gamma}), \sigma^{1/\gamma})$.

Under some assumptions we establish existence and uniqueness of solution $(a(t), h(t), u(x,t))$ for the problem (1)-(5) which belongs to the space $C[0,T] \times C^1[0,T] \times C^{2,1}(Q_T) \cap C(\overline{Q}_T)$ such that $a(t) > 0, h(t) > 0$ when $t \in [0,T]$.