A positivity principle for parabolic integro-differential equations and final overdetermination

Jaan Janno and Kairi Kasemets

Tallinn University of Technology

We prove a positivity principle for solutions of parabolic integrodifferential equations governing heat flow with memory. The proof is essentially based on the usage of the resolvent k of the memory kernel m. Namely, we assume $k \geq 0$ and $k' \leq 0$. Other assumptions are in case $k \neq 0$ weaker than in case $k = 0$. For instance, for the source term χ of the parabolic integro-differential equation it is sufficient to assume $\chi + k * \chi \geq 0$, where $*$ is the time convolution. This assumption is weaker than the assumption $\chi \geq 0$ that occurs in the case of the usual parabolic differential equation. Physically, this is explained by the inertia of the medium. The same remark holds for the second kind boundary condition, too.

Making use of the positivity principle we study some inverse problems with final overdetermination of the temperature. Namely, we prove existence, uniqueness and stability theorems for a linear inverse source problem and two nonlinear inverse coefficient problems for parabolic integro-differential equations. Existence and stability results for coefficient problems are local.

Our method works only in case the memory kernel m is independent of spatial variables. Otherwise it falls into a divergence-nabla type operator and time resolvent is not applicable.