Some uniqueness results for parameter identification in nonlinear hyperbolic PDEs

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The subject of this talk is motivated by applications in piezoelectric material characterization, which leads to the problem of identifying coefficient functions in nonlinear hyperbolic PDEs.

By means of the model problem of determining c in

$$u_{tt} - (c(u_x)u_x)_x = f \text{ in } \Omega = (0,1)$$

from overdetermined boundary measurements

$$c(u_x)u_x = g\,, \quad u = m \quad \text{ on } \partial\Omega = \{0,1\}$$

of u, in this talk we will discuss several approaches for showing uniqueness and stability.

The first one is to adapt the idea of integration along characteristics of the hyperbolic PDE [1] and leads to a stability conjecture of the form

$$\|c - \tilde{c}\|_{L^2} \leq C\|m - \tilde{m}\|_{H^1(0,T)}\,,$$

under appropriate assumptions, cf. [2].

The second approach works with a neighboring identification problem for which a sufficient condition for identifiability can directly be formulated and checked in terms of the given initial and boundary data. Assuming sufficient regularity of the PDE solution, can prove stability in the form

$$\|c - \tilde{c}\|_{L^\infty} \leq C\|g - \tilde{g}\|_{L^\infty(0,T)}\,,$$

and Lipschitz stable dependence also on the initial and source data in appropriate norms. An analogous result can be shown to hold true also in three space dimensions for coefficient functions of arbitrary finite dimension, cf. [3].

A third possibility arises when excitations at different intensities can be carried out and corresponding measurements can be taken. In that case and with a polynomial coefficient function c, an asymptotic expansion of u in therm of the excitation intensity enables a uniqueness result, cf. [4].

References

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