Uniqueness in nonlinearly coupled PDE systems

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Consider in $\Omega \times (0,T)$, $\Omega \subset \mathbb{R}^N$, the system

$$\theta_t - \Delta\theta = r(\theta, c) \tag{1}$$

$$c_t - \operatorname{div}(D(\theta,c)\nabla c) = 0 \tag{2}$$

of heat balance and carbon diffusion equations, complemented with nonhomogeneous boundary conditions on $\partial\Omega$

$$\frac{\partial\theta}{\partial\nu} + h(x)(\theta - \theta_\Gamma(x,t)) = 0 \tag{3}$$

$$-D(\theta,c)\frac{\partial c}{\partial\nu} = b(x,t), \tag{4}$$

and initial conditions for θ and c, as a model problem arising in steel hardening processes. Here, θ represents the absolute temperature, c is the carbon concentration, and $r(\theta,c)$, $D(\theta,c)$, $h(x)$, $\theta_\Gamma(x,t)$, and $b(x,t)$ are given functions. We mainly focus on the proof of uniqueness and Lipschitz continuous data dependence, which is based on a variant of the "L^p-Gronwall" inequality (joint work with Lucia Panizzi).