NULL CONTROLLABILITY PROPERTIES OF SOME DEGENERATE PARABOLIC EQUATIONS

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Motivated by several problems, e.g. the Crocco equation in fluid dynamics, the Fleming-Viot problem in biology, the Black-Scholes equation in economics, we are interested in controllability properties of parabolic equations degenerating at the boundary of the space domain. Typical models of such problems are:

- the following one dimensional case

$$u_t - (a(x)u_x)_x = f(x,t)\chi_{(a,b)}(x), \quad x \in (0,1), t > 0, \tag{0. 1}$$

where the control region (a,b) is a subdomain of $(0,1)$ and the function $a : [0,1] \to \mathbb{R}_+$ is continuous, of classe C^1 on $(0,1)$, and $a(0) = 0 = a(1)$;

- the N-dimensional case:

$$u_t - \text{div } (A(x)\nabla u) = f(x,t)\chi_\omega(x), \quad x \in \Omega, t > 0, \tag{0. 2}$$

where the control region ω is a subdomain of Ω, and the matrix $A(x)$ is definite positive for all $x \in \Omega$, and but has at least one eigenvalue equal to 0 for all $x \in \partial\Omega$.

This talk is focused on the N-dimensional case. Mainly, we assume that

- the least eigenvalue of the matrix $A(x)$ behaves as $d(x, \partial\Omega)^\alpha$, where $d(x, \partial\Omega)$ is the distance between x and the boundary of the domain Ω, and $\alpha \geq 0$,
- the degeneracy occurs in the normal direction: the associated eigenvector is the unit outward vector when $x \in \partial\Omega$.

Then three different cases have to be distinguished: (i): $\alpha \in [0,1)$, (ii): $\alpha \in [1,2)$, (iii): $\alpha \geq 2$.

In cases (i) and (ii), we derive new Carleman estimates for the adjoint degenerate parabolic equation, and the null controllability of the original problem; the proof of the Carleman estimates is based on some suitable Hardy type inequalities, and on the sharp regularity properties of the solution of the degenerate problem, studied in particular in the thesis of Dario Rocchetti (2008).

In case (iii), we prove that the problem is not null controllable, using earlier results of S. Micu and E. Zuazua (2001), L. Escauriaza, G. Seregin, V. Šverák (2004) concerning the null controllablity properties of the nondegenerate parabolic equations in unbounded domains.

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