THE CAGINALP PHASE-FIELD SYSTEM WITH SINGULAR POTENTIALS AND DYNAMIC BOUNDARY CONDITIONS

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Our aim in this talk is to discuss the well-posedness and the asymptotic behavior of the Caginalp phase-field system with a singular potential and dynamic boundary conditions,

$$\begin{cases} \eta \frac{\partial w}{\partial t} - \Delta w = -\frac{\partial u}{\partial t},\ \eta > 0, \\ \delta \frac{\partial u}{\partial t} - \Delta u + f(u) = w,\ \delta > 0, \\ \dfrac{\partial w}{\partial \nu}\Big|_{\Gamma} = 0, \quad u|_{\Gamma} = \psi, \\ \frac{\partial \psi}{\partial t} - \Delta_{\Gamma} \psi + \lambda \psi + g(\psi) + \dfrac{\partial u}{\partial \nu} = 0 \text{ on } \Gamma,\ \lambda > 0, \end{cases}$$

in a bounded and smooth domain $\Omega \subset \mathbb{R}^3$ with boundary Γ. Here, w denotes the temperature and u denotes the order parameter; this system was proposed by G. Caginalp in order to model melting-solidification phenomena in certain classes of materials. Furthermore, Δ_{Γ} is the Laplace-Beltrami operator and ν is the unit outer normal to Γ. Finally, f is singular, say, at -1 and 1 (typically, we have in mind the following thermodynamically relevant potential:

$$f(s) = -2\kappa_0 s + \kappa_1 \ln \frac{1+s}{1-s},\ s \in (-1,1),\ 0 < \kappa_0 < \kappa_1)$$

and g is regular.

These dynamic boundary conditions have been proposed by physicists, in the context of the Cahn-Hilliard equation for phase separation (note that, if we take $\eta = \delta = 0$, then we recover the Cahn-Hilliard equation), in order to account for the interactions with the walls in confined systems.

The main difficulty here is to prove proper separation properties (and corresponding dissipative estimates).