

# Observations on the moment problem and controllability for the Gurtin-Pipkin equation

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Gurtin-Pipkin equation has been introduced to take into account memory effect in heat processes. It is the equation

$$\theta'(t) = \int_0^t N(t-s)\theta(s) ds \quad (\mathbf{G-P})$$

where  $\theta = \theta(t, x)$ , where  $t > 0$  and  $x \in \Omega$ , a suitable region of  $\mathbf{R}^n$ .

When supplied with an initial condition  $\theta(0) = \xi$  and suitable boundary conditions, this equation identifies a well posed problem.

When a control acts on Equation **G-P**, either distributed or boundary control, we can put a control problem, i.e. we can ask to hit a prescribed target in a given time  $T$ . This problem has been studied by several authors. The proofs on these papers, based on inverse inequalities and Carleman estimates (see [1, 2]) or compactness arguments (see [3]) are not constructive. This fact, and the fact that when  $N(t) = 1$  Eq. **(G-P)** is an integrated form of the wave equation, suggest that we consider the special case  $\Omega = (0, \pi)$  and we try a constructive approach to the controllability problem in the case the control acts on the boundary,

$$\theta(t, 0) = u(t), \quad \theta(t, \pi) = 0.$$

We shall present several observation on the applicability of the moment problem to the solution of this control problem.

## References

- [1] Barbu, V. and Iannelli, M., Controllability of the heat equation with memory. *Differential Integral Equations*, 13 (2000), 1393–1412.
- [2] Yong, Jiongmin; Zhang, Xu Exact controllability of the heat equation with hyperbolic memory kernel. in “Control theory of partial differential equations”, 387–401, Lect. Notes Pure Appl. Math., 242, Chapman & Hall/CRC, Boca Raton, FL, 2005.
- [3] Pandolfi, Luciano The controllability of the Gurtin-Pipkin equation: a cosine operator approach. *Appl. Math. Optim.* 52 (2005), 143–165.