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On the Cahn-Hilliard equation with singular potential and dynamic boundary conditions

In this talk we will present some recent results, obtained in collaboration with G. Gilardi and A. Miranville, concerning existence, uniqueness, regularity properties, and long time behavior of solutions to the following equation of Cahn-Hilliard type:

$$u_t - \Delta(-\Delta u + f(u)) = g, \qquad (1)$$

where the unknown u lives in a smooth bounded domain $\Omega \subset \mathbf{R}^3$ for times $t \in (0, +\infty)$, g is a volumic source, and no-flux boundary conditions are assumed for the function $w = -\Delta u + f(u)$ (chemical potential).

The main novelty of our problem is represented by the *dynamic* boundary condition assumed for u, namely

$$v_t - \Delta_\Gamma v + f_\Gamma(v) = -\partial_{\mathbf{n}} u, \qquad (2)$$

where $\Gamma := \partial\Omega$, $v := u|_\Gamma$, Δ_Γ is the Laplace-Beltrami operator on Γ, and $\partial_{\mathbf{n}}$ denotes the (outer) normal derivative.

The system (1-2), coupled with suitable Cauchy conditions, will be studied in case one or both of the nonlinearities f, f_Γ are *singular* functions. This means that f and f_Γ are increasingly monotone up to a linear perturbation, but they could be defined only inside a bounded interval of the real line (that consisting of the physically admissible values of u). A meaningful example is given by the so-called *logarithmic* nonlinearity

$$f(u) = (1-u)\log(1-u) + (1+u)\log(1+u) - \lambda u^2, \qquad \lambda \geq 0,$$

which makes sense for $u \in (-1, 1)$.

We will show that, under rather weak compatibility conditions between f and f_Γ, a unique weak solution exists. Moreover, we will investigate the long-time properties of the semiflow associated to this problem from the point of view of ω-limits of trajectories as well as of global and exponential attractors.