

HARDY'S INEQUALITIES AND SINGULAR INVERSE-SQUARE POTENTIALS IN CONTROL THEORY

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From the works of Baras and Goldstein [1, 2], it is well-known that singular inverse-square potentials, arising, for example, in the context of combustion theory and quantum mechanics, generate interesting phenomena: when one replaces the Laplace operator $-\Delta$ by $-\Delta - \lambda|x|^{-2}I$ in the heat equation, (global) existence and non-existence (instantaneous blow-up) of positive solutions is crucially determined by the value of the parameter λ , the critical value of λ being the constant $\lambda^* := (N - 2)^2/4$, that appears in the so-called Hardy inequality.

We present here the results of two joint works with E. Zuazua in which we address the question of null (or exact) controllability of heat, wave and Schrödinger equations perturbed by such inverse-square potentials. Null controllability is proved in the range of sub-critical coefficients $\lambda \leq \lambda^*$ and under suitable geometric conditions. The proofs of the observability inequalities rely on Carleman estimates [4, 3] or on the method of multipliers [5], the key point being each time a suitable Hardy-type inequality. On the contrary, in the super-critical case, null controllability $\lambda > \lambda^*$ becomes false [3, 5].

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