AN INVERSE PROBLEM RELATED TO A DEGENERATE PARABOLIC INTEGRODIFFERENTIAL EQUATION

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We recover a memory kernel k depending on time and one spatial variable x_3 in a linear degenerate parabolic second-order integrodifferential equation related to an unbounded cylinder $\omega \times \mathbf{R}$, where ω is a star-shaped bounded open subset of $\mathbf{R}^2$. More explicitly, x_3 denotes the axial variable and the second-order differential operator entering the equation is assumed to degenerate at ∞.

The identification problem we deal with is of some interest in applied problems related, e.g., to *stratified* media. Indeed, when the convolution kernel k depends on time and space variables and the degeneracy m does not depend on time, the equation governing a thermal body Ω_1 with memory takes the following general form, where $(t,x) \in [0,T] \times \Omega_1$:

$$D_t[m(x)u(t,x)] = \operatorname{div} \mathcal{E}u(t,x) + \operatorname{div} \int_0^t k(t-s,\mu(x))\mathcal{F}u(s,x)\,\mathrm{d}s + f(t,x).$$

Here, $\mathcal{E}$ and $\mathcal{F}$ are two linear first-order operators with coefficients depending on x, only, $\operatorname{div} \mathcal{E}$ is a uniformly elliptic operator, $\mu : \Omega_1 \to \mathbf{R}$ is a given function ruling the spatial dependence of k, and f represent the system of external heat sources.