Derivation and Applications of Dynamic Boundary Conditions to Nonlinear Partial Differential Equations

We give a unified derivation of the heat equation and other equations with dynamic boundary conditions on a bounded region $\Omega \subseteq \mathbb{R}^n$ which are incorporated into the derivation of the equation. For the heat equation we consider dynamic boundary conditions of the form

$$\frac{\partial u}{\partial t} - a(x)\,\Delta_{LB} u + b(x)\,\frac{\partial u}{\partial n} + c(x)\,u \in -\beta(x,u)\,,$$

where β is a maximal monotone graph and $a, b, c \in C(\partial\Omega)$, $a(x) \geq 0$, and $b(x) > 0$. We obtain well-posedness and regularity results. Our regularity results are analogous to those of H. Brezis and L.C. Evans (who worked with boundary conditions of the form $\frac{\partial u}{\partial n} \in -\beta(x,u)$) and correspond to the optimal results suggested by the linear hypercontractivity estimates. We mention some recent results for the Cahn-Hillard equation with dynamic boundary conditions. Finally we present new results in which dynamic boundary conditions play a fundamental role in obtaining significant extensions of the classical results of Landisman and Lazar.

The works to be discussed are partly in collaboration with Angelo Favini (Università di Bologna), Ciprian Gal (Univeristy of Memphis), Jerry Goldstein (Univeristy of Memphis), and Silvia Romanelli (Università di Bari).