Two integrodifferential identifications problems

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Problem 1. *(in co-operation with E. Sinestrari.)* Consider the following second-order integrodifferential problem related to a Banach space H: *find two functions* $a : [0, T] \to \mathbb{R}$ *and* $u : [0, T] \to H$ *solutions to*

$$(P1) \quad \begin{cases} u''(t) - Au(t) = \displaystyle\int_0^t a(t-s)A_1 u(s)\, ds + f(t), & t \in [0, T], \\ u(0) = u_0, \qquad u'(0) = u_1, & \\ \langle \Phi, u(t) \rangle = g(t), & t \in [0, T], \end{cases}$$

where $A : D(A) \subset H \to H$ is the generator of a cosine function, $A_1 \in \mathcal{L}(D(A), H)$, $\Phi \in H^*$, the dual space to H, and $f : [0, T] \to H$, $g : [0, T] \to \mathbb{R}$ and $u_0, u_1 \in H$ are given.
Usually problem (P) is solved (locally or globally in time) when the unknown kernel a is sought for in $W^{1,p}((0, T))$ for some $p \in (1, +\infty]$.
The aim of this talk is to generalize the space where unknown a is sought for to $C([0, T]) \cap BV([0, T])$, i.e to a space where functions need not to be *absolutely continuous*.
This generalization strictly depends on a new estimate by E. Sinestrari involving convolutions $a * f(t) = \int_0^t a(t-s)f(s)\, ds$, where $a \in C([0, T]) \cap BV([0, T])$ and $f \in C([0, T]; H)$.
A general existence and uniqueness result for the solution to problem (P) can be proved. Such a result can be applied to hyperbolic integrodifferential problems of type (P) related to open bounded domains Ω of class C^2, in the context of Sobolev spaces related to $L^2(\Omega)$.

Problem 2. *(in co-operation with G. Di Blasio.)* Consider the following delay functional differential problem related to a Banach space E: *find two functions* $f : [0, T] \to \mathbf{R}$ *and* $u : [0, T] \to E$ *solutions to*

$$(P2) \quad \begin{cases} u'(t) = Au(t) + aAu(t-r) + \displaystyle\int_{-r}^0 b(s)Au(t+s)\, ds + f(t)z, & \text{for a.e. } t \in (0, T), \\ u(s) = \varphi(s), \quad \text{for a.e. } s \in (-r, 0), \qquad u(0) = \varphi_0, & \\ \Phi[u(t)] = g(t), & t \in [0, T]. \end{cases}$$

Here A is the infinitesimal generator of an analytic semigroup in a Banach space E, $a \in \mathbb{R}$, and Φ is a linear functional defined on $D(\Phi) \subseteq E$. Further, $b : (-r, 0) \to \mathbb{R}$, $z \in E$, $\varphi : (-r, 0) \to E$ and $\varphi_0 \in E$ and $g : [0, T] \to \mathbb{R}$ are given. Here we do not require that $D(\Phi) = E$: this will allow, in the case where A arises from elliptic operators on Ω, to consider also conditions given on the boundary and/or involving the derivatives of u.
Concerning the relations between the datum z and the functional Φ in (P2), possible situations are the following

$$\Phi[z] \neq 0 \quad \text{or} \quad \Phi[A^j z] = 0,\ j = 0, \ldots, m-1, \quad \Phi[A^m z] \neq 0.$$

The first assumption will be referred to as the *nonsingular case*, whereas the latter will be called the *singular case*.

For such problems existence and uniqueness results as well as continuous dependence upon the data are proved. Applications to partial differential equations, with measurement of the flux on the boundary, are given.