Well-posedness results for a fast diffusion model

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We consider a general class of nonlinear degenerate elliptic parabolic problems associated with the equation

$$\frac{\partial C^*(h)}{\partial t} - \nabla \cdot (k(h)\nabla h) + \frac{\partial k(h)}{\partial x_3} = f \text{ in } \Omega \times (0,T),$$

with initial and boundary data. Under certain hypotheses this equation may degenerate ($\frac{dC^*(h)}{dh} = 0$ for $h \geq 0$) and by suitable function transformations it can be written in the equivalent diffusive form

$$\frac{\partial \theta}{\partial t} - \Delta\beta^*(\theta) + \frac{\partial K(\theta)}{\partial x_3} = f \text{ in } \Omega \times (0,T),$$

where β^* is a multivalued function defined for $\theta \leq \theta_s < \infty$. We shall consider certain analytical properties of the function β^* which embed this equation in the classes of slow and fast diffusion, the latter being characterized by a blowing up diffusivity coefficient at a finite value of the solution (in our case at $\theta = \theta_s$). In particular such equations describe free boundary processes associated to fluid flow in saturated-unsaturated porous media.

We are concerned with the proof of the well-posedness of the model (especially for the strongly nonlinear case) developed in the framework of the theory of evolution equations with m-accretive operators in Hilbert spaces.