A new identification problem with evolution on the boundary *

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The problem we are concerned with consists in identifying the unknown (time) convolution scalar kernel k appearing in the linear second–order elliptic equation

$$A(u + k * u) = f \qquad \text{in} \quad (0,T) \times \Omega, \tag{0.1}$$

where $k * u(t,x) = \int_0^t k(t-s)u(s,x)\,ds$ and Ω is an open bounded subset in $\mathbf{R}^n$.
Equation (0.1) is not endowed with the usual Dirichlet or Neumann boundary conditions, but with dynamical ones, i.e.

$$D_t u + bD_\nu u = g, \qquad \text{on} \quad (0,T) \times \partial\Omega. \tag{0.2}$$

We determine $u : [0,T] \times \Omega \to \mathbf{R}$ and $k : [0,T] \to \mathbf{R}$ satisfying (0.1), (0.2) and the following conditions

$$u(0,\cdot) = u_0, \quad \text{on } \partial\Omega, \tag{0.3}$$

$$\Phi[u(t,\cdot)] = l(t), \quad t \in [0,T], \tag{0.4}$$

where ν denotes the conormal unit vector related to A and $\Phi \in L^p(\partial\Omega)^*$, e.g.,

$$\Phi[z] = \int_\Gamma \varphi(x)z(x)\,d\sigma(x), \tag{0.5}$$

Γ being an open subset of $\partial\Omega$ having a positive surface measure.
A global existence and uniqueness of the solution of the problem is proved.

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