Toric arrangements, matroids, polytopes.

The purpose of this course is to make the students able to start doing research in a field that has been developing rapidly in the last years. (In this spirit, we will focus more on examples than on proofs!). This field lies at the crossroads of Combinatorics, Topology and Algebra: we will approach the combinatorial core of the theory and then, depending on the background and interests of the audience, the algebraic and geometric import will be developed or only sketched, and connections with other disciplines (Numerical analysis, Economics...) will be outlined or not.

The course will be divided in four parts:

1 HYPERPLANE ARRANGEMENTS
Definition; the intersection poset and characteristic polynomial; relations with the Poincaré polynomial. The Orlik-Solomon algebra. Zonotopes. Examples of hyperplane arrangements:
- Coxeter arrangements (relations with Lie algebras or with Artin groups may be outlined).
- Arrow's impossibility theorem in social choice theory;
- graphic arrangements; proper colorings are counted by the characteristic polynomial.

2 MATROIDS AND TUTTE POLYNOMIALS
Some of the equivalent definitions of a matroid; realizability. Duality for matroids and for graphs: chromatic and flow polynomials. Deletion-contraction. The Tutte polynomial. Positivity, Crapo's formula. Convolution formula. Basis polytope and applications.

3 TORIC ARRANGEMENTS AND ARITHMETIC TUTTE POLYNOMIALS
Definition of toric arrangements; the layers poset; examples. Relations with periodic hyperplane arrangements; the case of affine Coxeter arrangements. Characteristic and Poincaré polynomial. The arithmetic Tutte polynomial: applications to toric arrangements and zonotopes. Positivity, combinatorial interpretation. Arithmetic matroids; product of multiplicities; convolution. Wonderful compactifications for hyperplane arrangements and toric arrangements (sketch).

4 INTEGER POINTS IN POLYTOPES
Introduction to vector partition functions; applications (e.g., to representation theory). Integer points in polytopes as a discrete analogue of the volume; splines and approximation theory. Ehrhart polynomials. Relations of splines and vector partition functions with hyperplane arrangements and toric arrangements (via Laplace transform). Piecewise polinomiality and quasi-polynomiality, Dahmen-Micchelli spaces.