MECCANICA

A CONFERENCE IN HONOR OF SANDRO GRAFFI ON HIS 65TH BIRTHDAY

ABSTRACTS

Bologna, August 27-30, 2008

Aula Absidale Santa Lucia

Jean Bellissard
(Georgia Institute of Technology, USA)

The topology of tiling spaces

Tilings have been studied for a long time due to their fascinating beauty and symmetry properties. In the nineties, they were related to the description of aperiodic solids with long range order for the atomic position. However it is only during the last fifteen years that topology has been introduced to investigate some of their global properties. This review talk will introduce the notion of *Hull* or *Tiling Space*, and will focus on an important subclass of tilings, that are *repetitive* and have *finite local complexity* (FLC). It will describe various topological invariant such as its C^*-algebra, its K-group, and various equivalent definition of its cohomology. It will provide the latest results obtained to compute such invariants and give the so-called gap labeling theorem in full generality.

Part of this talk concerns a recent joint work with J. Savinien (GeorgiaTech).

Francesco Calogero
(Sapienza Università di Roma, Italy)

Isochronous systems are not rare

A (classical) dynamical system is called *isochronous* if it features an *open* (hence *fully dimensional*) region in its phase space in which *all* its solutions are *completely periodic* (i. e., periodic in *all* degrees of freedom) with the *same* fixed period (independent of the initial data, provided they are inside the *isochrony* region). When the isochrony region coincides with the *entire* phase-space one talks of *entirely isochronous* systems. A trick is presented associating to a dynamical system a modified system depending on a parameter so that when this parameter vanishes the original system is reproduced while when this parameter is *positive* the modified system is *isochronous.* This technique is applicable to large classes of dynamical systems, justifying the title of this talk. An analogous technique (introduced with F. Leyvraz), even more widely applicable — for instance, to *any* translation-invariant (classical) many-body problem — transforms a real autonomous *Hamiltonian* system into an *entirely isochronous* real autonomous *Hamiltonian* system. The modified system is of course no more translation-invariant, but in its centre-of-mass frame it generally behaves quite similarly to the original system over times much shorter than the isochrony period T (which may be chosen at will). Hence, when this technique is applied to a "realistic" many-body Hamiltonian yielding, in its centre of mass frame, *chaotic* motions with a natural time-scale much smaller than (the chosen) T, the corresponding modified Hamiltonian shall yield a chaotic behavior (implying statistical mechanics, thermodynamics with its second principle, etc.) for quite some time before the *entirely isochronous* character of the motion takes over hence the system returns to its initial state, to repeat the cycle over and over again. We moreover show that the quantized versions of these modified Hamiltonians feature infinitely degenerate *equispaced* spectra. Analogous techniques are applicable to nonlinear evolution PDEs, but in this talk there will be no time to cover this aspect. The material presented is a synthesis of work done over the last 10 years with several collaborators, as reviewed in my 264-page monograph entitled *Isochronous systems*, just published (February 2008) by Oxford University Press. Perhaps some new (more recent) results will also be presented.

Francesco Cannata
(INFN, Bologna, Italy)

Scattering in PT-symmetric Quantum Mechanics

After a brief prelude concerning my relation to Sandro Graffi, I discuss scattering in PT-symmetric one-dimensional Quantum Mechanics within the Schrödinger and Dirac framework. In addition to standard local finite range potentials, also non local separable potentials will be considered.

Jean-Michel Combes
(Université du Sud, Toulon-Var, France)

Eigenvalue statistics for random Schrödinger operators

We present various estimates on the number of eigenvalues for the Anderson model in the discrete and continuous case. Emphasis is on the Wegner and Minami estimates which play important rôles in localization theory and spectral analysis for this model. They imply in particular absence of energy level repulsion in the strong localization regime; this follows from the Poisson distribution of properly rescaled eigenvalues with an intensity measure given by the density of states.

Jürg Fröhlich
(ETH Zurich, Switzerland)

Quantum Brownian Motion, Quantum Friction and decoherence

I will discuss some recent results in the 'Quantum Theory of Experiments', including the following ones:

(1) Quantum Brownian Motion of a heavy quantum particle interacting with quantum-mechanical heat baths;
(2) Hamiltonian and Quantum Friction for particles interacting with light or moving through a Bose-Einstein condensate;
(3) Decoherence for these systems.

It is argued that these results clarify what it is that quantum theory enables us to predict about the outcome of experiments.

Cristian Giardinà
(Technische Universiteit, Eindhoven, The Netherlands)

Duality and hidden symmetries in transport models

The transformation to dual processes is a technique developed in the probabilistic literature (Liggett) that allows one to obtain elegant and general solutions of some interacting particle systems. One transforms the evaluation of a correlation function in the original process to a simpler quantity in the dual one. Some years ago, Sandow and Schutz recognized that non-abelian symmetries in the evolution operator naturally yield dual models. They exemplified this with the simple symmetric exclusion process whose $SU(2)$ symmetry they made explicit by writing the evolution operator in quantum spin notation. We will consider duality for transport models that

(i) have in the bulk a symmetry associated with a conserved quantity (the one that is transported);
(ii) are coupled to reservoirs at their boundaries.

For the transport of mass we consider the $SU(2)$ symmetry for generalized exclusion processes that allow up to $2J$ particles per site. This generalizes previous results and helps to understand the role of boundaries for the dual process in the easy context of finite state space. Then we consider Markov processes with a continuous state space. For models of energy transport we uncover a hidden $SU(1,1)$ symmetry. We discuss general strategies to find out whether a model admits a dual.

Italo Guarneri
(Università dell'Insubria, Italy)

Aspects of kicked quantum dynamics

Kicked quantum dynamics is a denotation of discrete-time quantum dynamical systems. More than thirty years after the invention of the Kicked Rotor, it is still in the focus of active research. On the one hand, it has given birth to an ever increasing list of variants of the basic original prototypes, which have provided formally simple models for investigation of some general properties of quantum transport: these include dynamical localization, anomalous diffusion, decay from stable phase-space islands, and lately directed transport. On the other hand, renewed interest on the physical side has been stimulated by experimental realizations, which are now possible, thanks to the science and technology of cold and ultra-cold atoms. However, quite a few exact results have been obtained so far. In this talk some mathematical problems, connected with recent models of kicked quantum dynamics, will be reviewed.

Francesco Guerra
(Sapienza Università di Roma, Italy)

Structure and properties of the Generalized Random Energy Model

We recall the general structure and properties of the Generalized Random Energy Model, introduced by Bernard Derrida in order to provide a simple model with properties similar to a spin glass. This will be an occasion to review comparison methods and interpolation techniques, that have been exploited in the last years in the study of spin glass models. Moreover, the new variational principles, extending those of Parisi type for the spin glass, find a completely well defined and simple formulation in the quite simple case of the Generalized Random Energy Model.

STEFANO ISOLA
(UNIVERSITÀ DI CAMERINO, ITALY)

ORDERINGS OF THE RATIONALS: DYNAMICAL SYSTEMS AND STATISTICAL MECHANICS

After recalling the basic properties of the Stern-Brocot tree and its connection with continued fractions, we show that (a permuted version of) it can be constructed dynamically in two different ways, which in turn are related to the geodesic and horocycle flows on the modular surface, respectively. We then briefly discuss some further connections with the spectral theory of the hyperbolic Laplacian, the statistical mechanics of a class of ferromagnetic spin chains and some stochastic processes on the positive real line.

GIOVANNI JONA-LASINIO
(SAPIENZA UNIVERSITÀ DI ROMA, ITALY)

LONG RANGE CORRELATIONS IN DIFFUSIVE SYSTEMS

Long range correlations are a generic feature of nonequilibrium steady states and have been observed esperimentally. This phenomenon can be demonstrated in microscopic models but can also be derived from simple postulates characterizing the macroscopic behavior of diffusive systems. One of the postulates is the Einstein relationship among transport coefficients. A general theorem then excludes these correlations in equilibrium states, even if inhomogeneous such as, e.g., sedimentation equilibrium in centrifugal or gravitational fields. There exist, however, as pointed out by Joel Lebowitz, David Mukamel and others, some equilibrium models that exhibit long range correlations. A simple analysis shows that in these cases the Einstein relationship fails.

Joel Lebowitz
(Rutgers University, New Brunswick, USA)

Exact results for ionization of model atomic systems

We present rigorous results for the ionization of model quantum systems with reference Hamiltonian $H_0 = -\frac{1}{2}\nabla^2 + V_0(x)$ (with $x \in \mathbb{R}^d$) having both bound and continuum states subjected to arbitrary strength time-periodic potentials $V_1(x,t) = V_1(x, t+2\pi/\omega)$. Starting from an initially localized state $\psi_0(x)$, we prove, for a large class of $V_0(x)$ and $V_1(x,t)$, that the wavefunction $\psi(x,t)$ will delocalize as $t \to \infty$, i.e. the system will ionize. The only exceptions are cases where there are time-periodic bound states of the Floquet operator associated to H_0+V_1. These do occur (albeit rarely) when V_1 is not small. Proof of ionization then involves showing that the Floquet operator has only absolutely continuous spectrum. For small V_1 and compact V_0, we prove convergence of the perturbation expansion for the resonances (defined as poles of an appropriate resolvent) $\sigma_j = E_j - i\Gamma_j/2$ (with E_j the resonance energy and Γ_j the ionization rate), justifying Fermi's golden rule. For very long times $\psi(x,t)$ is given (for compact V_0) by a power series in $t^{-1/2}$ which we prove in some cases to be Borel summable. For the Coulomb potential $V_0(x) = -b\,|x|^{-1}$ in $\mathbb{R}^3$, we prove ionization for $V_1(x,t) = V_1(|x|)\sin(\omega t)$, $V_1(|x|) = 0$ for $|x| > R$ and $V_1(|x|) > 0$ for $|x| < R$. If ψ_0 is compactly supported both in x and in angular momentum, $\mathbf{L}$, we obtain that $\psi(x,t) \sim O(t^{-5/6})$ as $t \to \infty$.

(Joint work with O. Costin, C. Stucchio and S. Tanveer.)

Giorgio Parisi
(Sapienza Università di Roma, Italy)

On the most compact regular lattices in large dimensions

In this talk I will consider the computation of the maximum density of regular lattices in large dimensions using an approach based on statistical mechanics. The starting point will be some theorems of Rogers, which are virtually unknown in the community of physicists. Using his approach one can find many similarities (and differences) with the problem of computing the entropy of a system of hard spheres. The relation between the two problems is investigated in detail. Some conjectures are presented: further investigation is needed in order to check their consistency.

Thierry Paul
(Ecole Normale Superieure, Paris, France)

Long time semiclassical evolution

We will present some recent results concerning the long time quantum evolution in the semiclassical limit, that is, uniform semiclassical approximation for times diverging as the Planck constant tends to zero. We will present results for both propagation of observables and coherent states in the general situation, and will emphasise several particular cases associated to stable and unstable underlying dynamics for which the time-scales can be improved.

Mario Pulvirenti
(Sapienza Università di Roma, Italy)

Mean-Field limit and semiclassical expansion for an N-particle quantum system

We consider a N-particle quantum system interacting via a mean-field Hamiltonian. As $N \to \infty$, the one-particle state obeys the Hartree equation and the propagation of chaos holds. In this paper we analyze the dependence by proving that each term of the semiclassical expansion of the N-particle system agrees, in the limit, with the corresponding term associated to the Hartree equation. We work in the classical phase space by using the Wigner formalism which seems the most appropriate for the problem at hand.

(Joint work with F. Pezzotti.)

Harris J. Silverstone
(Johns Hopkins University, Baltimore, USA)

Rereading Langer's influential 1937 JWKB paper: the unnecessary Langer transformation; the ambiguous $\hbar$; the power of Borel summation

In the Abstract of his influential 1937 paper, Langer attacked the JWKB analysis of the radial wave equation as "uncritical and in error." Langer "justified" Kramers' empirical substitution of $(l+\frac{1}{2})^2$ for $l(l+1)$ in the centrifugal potential, which converted the behavior of the JWKB wave function at the origin from $r^{\frac{1}{2}+[l(l+1)]^{1/2}}$ to r^{l+1}, via the transformation $r = e^x, \psi(r) = e^{x/2}u(x)$, which moved the origin to $-\infty$ and eliminated the second-order pole. The first-order-in-$\hbar$ x formulas turned out to be equivalent to first-order-in-$\hbar$ r formulas with $l(l+1)$ replaced by $(l+\frac{1}{2})^2$. Krieger and Rosenzweig (1967) pushed Langer's x solution to third order but concluded that "there is no effective [r] potential ... which will give rise to the correct result ..." Beckel and Nakhleh (1963) found a value K to substitute for $l(l+1)$ to fix the third-order (but not first-order) JWKB wave function. Fröman and Fröman (1974) extended that to 9th, and Seetharaman and Vasan (1984) to any order. In infinite order K is exactly $l(l+1)$, i.e., no modification. Hainz and Grabert (1999) got r^{l+1} in all orders beyond zeroth by the decomposition, $\hbar^2 l(l+1) = L^2 + \hbar L$ (at the end, L is set equal to $\hbar l$), with no Langer modification. The big surprise came in 2004 when Dahl and Schleich saw that Langer's exponential transformation was irrelevant, that only $r^{-1/2}$ was relevant, and that "[Langer's] analysis may, in fact, be considered as nothing more than a somewhat complicated way of solving [the radial wave equation for $r^{-1/2}\psi(r)$] by the JWKB method." These seemingly incompatible results can be understood in a unified framework by noting that for each there are *two* $\hbar$'s: the kinetic energy $\hbar$ drives the expansion; the centrifugal potential $\hbar$ is passive, implicit, intrinsic, never expanded. The solutions differ in how the "original $\hbar$" is split between expansion $\hbar$ and implicit $\hbar_i$. The different JWKB expansions are Borel summable to different analytic functions that coincide when $\hbar_i = \hbar$. In two-$\hbar$ notation, the generalization of Kramers' substitution is near trivial:

$$\left(-\frac{\hbar^2}{2}\frac{d^2}{dr^2} + V(r) + \frac{\hbar_i^2(l+\frac{1}{2})^2}{2r^2} - \frac{\hbar^2(\frac{1}{2})^2}{2r^2} - E\right) e^{\int q dr/\hbar} = 0.$$

Giorgio Turchetti

(Università di Bologna, Italy)

Theory and simulations for weakly chaotic systems: round off and irreversibility, collisions and relaxation

The theory of dynamical systems is suited to describe ordered, chaotic and weakly chaotic systems. The last ones include N body systems with long range interactions described by the Vlasov equation in the continuum limit $N \to \infty$. The collisional effects for finite N render the system weakly chaotic and drive it slowly to thermodynamic equilibrium. No basic theory is available to describe the relaxation and we must rely on numerical integration of Hamilton's equations. A phenomenological approach where the collisions are replaced by a Wiener noise has been proposed by Landau, and though ignoring the rare hard collisions, it provides correct scaling laws. Three problems are waiting for an answer: the charaterization of weak chaos where power law decays of correlations are observed, the description of dynamical effects introduced by finite precision computations based on round off arithmetics, the formulation of stocastic equations where the noise p.d.f. has the power law decay suggested by the time series obtained from N body simulations. Weakly chaotic toy models of low dinemsionality allow one to esplore the first two problems. The spectra of Poincaré recurrences show that the decay of correlations in a region of weak chaos is the same as at the boundary between an integrable and a mixing map. The reversibility error and the correlation decay show that the round off error is equivalent, to a large extent, to a random perturbation whose effect can be analytically explored. The relaxation to thermodynamic equilibrium for a model of N Coulomb oscillators (a 2D model with logarithmic potential) occurs in a time interval proportional to N and the p.d.f. of the momentum jump exhibits a power-law decay. Even though the replacement of the collisions with a random perturbation having the same p.d.f. makes the system irreversible, the actual N body dynamics transferred on a computer becomes irreversible due to the finite precision arithmetics (suppose we can neglect the discretization error we make by replacing Hamilton's equations with a symplectic map). The round off acts as a noise whose amplitude is b^{-m} where b is the base and m the number of significant digits. In the Langevin equations describing the collisional effects we can neglect the round-off noise, whereas in the N body dynamics we cannot, since it makes the system irreversible. The information on any physical system being finite (position and velocity accuracy are limited by atomic size and thermal motion), irreversibily pops out naturally without any *ad hoc* hypothesis like Bolzmann or Prigogine did. As a consequence, computer simulation is closer to physical reality than the Platonic description provided by Hamilton's equations in a Euclidean phase space, where the specification of any state reequires infinite information.

Claude Viterbo

(Ecole Polytechnique, France)

Symplectic Homogenization

We show that in a suitable sense the sequence $k \longrightarrow H(k \cdot q, p)$ of Hamiltonians converges to an integrable Hamiltonian $\overline{H}(p)$. We shall give a number of applications of this result, mostly to dynamics and symplectic geometry.

André Voros
(CEA-CENS, Saclay, France)

Toward a quantum-integrable structure of the general 1D Schrödinger equation

We review the exact WKB solution method for the 1D Schrödinger equation in the general polynomial-potential case. Analogies, with exactly solvable systems in lattice statistical mechanics and quantum field theory, suggest that our exact semiclassical solvability generalizes the latter completely integrable structures toward one we call quantum integrability; we see this as corresponding to the Liouville integrability (of 1D classical mechanics) at the quantum level. Compatibility with another major quantum structure, singular perturbation theory, requires a nontrivial analysis which is partially done.

Kenji Yajima
(Gakushuin University, Tokyo, Japan)

On the fundamental solution for Schrödinger equations

The smoothness and boundedness properties of the fundamental solution to the Schrödinger equation

$$i\partial_t u = (1/2)(-i\partial_x - A(t,x))^2 u + V(t,x)u$$

strongly depend on the behavior of $V(t,x)$ and $A(t,x)$ at infinity. I will survey recent results on this problem.