MECCANICA

A CONFERENCE IN HONOR OF SANDRO GRAFFI ON HIS 65TH BIRTHDAY

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Isochronous systems are not rare

A (classical) dynamical system is called *isochronous* if it features an *open* (hence *fully dimensional*) region in its phase space in which *all* its solutions are *completely periodic* (i. e., periodic in *all* degrees of freedom) with the *same* fixed period (independent of the initial data, provided they are inside the *isochrony* region). When the isochrony region coincides with the *entire* phase-space one talks of *entirely isochronous* systems. A trick is presented associating to a dynamical system a modified system depending on a parameter so that when this parameter vanishes the original system is reproduced while when this parameter is *positive* the modified system is *isochronous.* This technique is applicable to large classes of dynamical systems, justifying the title of this talk. An analogous technique (introduced with F. Leyvraz), even more widely applicable — for instance, to *any* translation-invariant (classical) many-body problem — transforms a real autonomous *Hamiltonian* system into an *entirely isochronous* real autonomous *Hamiltonian* system. The modified system is of course no more translation-invariant, but in its centre-of-mass frame it generally behaves quite similarly to the original system over times much shorter than the isochrony period T (which may be chosen at will). Hence, when this technique is applied to a "realistic" many-body Hamiltonian yielding, in its centre of mass frame, *chaotic* motions with a natural time-scale much smaller than (the chosen) T, the corresponding modified Hamiltonian shall yield a chaotic behavior (implying statistical mechanics, thermodynamics with its second principle, etc.) for quite some time before the *entirely isochronous* character of the motion takes over hence the system returns to its initial state, to repeat the cycle over and over again. We moreover show that the quantized versions of these modified Hamiltonians feature infinitely degenerate *equispaced* spectra. Analogous techniques are applicable to nonlinear evolution PDEs, but in this talk there will be no time to cover this aspect. The material presented is a synthesis of work done over the last 10 years with several collaborators, as reviewed in my 264-page monograph entitled *Isochronous systems*, just published (February 2008) by Oxford University Press. Perhaps some new (more recent) results will also be presented.