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EXACT RESULTS FOR IONIZATION OF MODEL ATOMIC SYSTEMS

We present rigorous results for the ionization of model quantum systems with reference Hamiltonian $H_0 = -\frac{1}{2}\nabla^2 + V_0(x)$ (with $x \in \mathbb{R}^d$) having both bound and continuum states subjected to arbitrary strength time-periodic potentials $V_1(x,t) = V_1(x, t + 2\pi/\omega)$. Starting from an initially localized state $\psi_0(x)$, we prove, for a large class of $V_0(x)$ and $V_1(x,t)$, that the wavefunction $\psi(x,t)$ will delocalize as $t \to \infty$, i.e. the system will ionize. The only exceptions are cases where there are time-periodic bound states of the Floquet operator associated to H_0+V_1. These do occur (albeit rarely) when V_1 is not small. Proof of ionization then involves showing that the Floquet operator has only absolutely continuous spectrum. For small V_1 and compact V_0, we prove convergence of the perturbation expansion for the resonances (defined as poles of an appropriate resolvent) $\sigma_j = E_j - i\Gamma_j/2$ (with E_j the resonance energy and Γ_j the ionization rate), justifying Fermi's golden rule. For very long times $\psi(x,t)$ is given (for compact V_0) by a power series in $t^{-1/2}$ which we prove in some cases to be Borel summable. For the Coulomb potential $V_0(x) = -b\,|x|^{-1}$ in $\mathbb{R}^3$, we prove ionization for $V_1(x,t) = V_1(|x|)\sin(\omega t)$, $V_1(|x|) = 0$ for $|x| > R$ and $V_1(|x|) > 0$ for $|x| < R$. If ψ_0 is compactly supported both in x and in angular momentum, $\mathbf{L}$, we obtain that $\psi(x,t) \sim O(t^{-5/6})$ as $t \to \infty$.

(Joint work with O. Costin, C. Stucchio and S. Tanveer.)