MECCANICA

A CONFERENCE IN HONOR OF SANDRO GRAFFI ON HIS 65TH BIRTHDAY

Giorgio Turchetti

(Università di Bologna, Italy)

Theory and simulations for weakly chaotic systems: round off and irreversibility, collisions and relaxation

The theory of dynamical systems is suited to describe ordered, chaotic and weakly chaotic systems. The last ones include N body systems with long range interactions described by the Vlasov equation in the continuum limit $N \to \infty$. The collisional effects for finite N render the system weakly chaotic and drive it slowly to thermodynamic equilibrium. No basic theory is available to describe the relaxation and we must rely on numerical integration of Hamilton's equations. A phenomenological approach where the collisions are replaced by a Wiener noise has been proposed by Landau, and though ignoring the rare hard collisions, it provides correct scaling laws. Three problems are waiting for an answer: the charaterization of weak chaos where power law decays of correlations are observed, the description of dynamical effects introduced by finite precision computations based on round off arithmetics, the formulation of stocastic equations where the noise p.d.f. has the power law decay suggested by the time series obtained from N body simulations. Weakly chaotic toy models of low dinemsionality allow one to esplore the first two problems. The spectra of Poincaré recurrences show that the decay of correlations in a region of weak chaos is the same as at the boundary between an integrable and a mixing map. The reversibility error and the correlation decay show that the round off error is equivalent, to a large extent, to a random perturbation whose effect can be analytically explored. The relaxation to thermodynamic equilibrium for a model of N Coulomb oscillators (a 2D model with logarithmic potential) occurs in a time interval proportional to N and the p.d.f. of the momentum jump exhibits a power-law decay. Even though the replacement of the collisions with a random perturbation having the same p.d.f. makes the system irreversible, the actual N body dynamics transferred on a computer becomes irreversible due to the finite precision arithmetics (suppose we can neglect the discretization error we make by replacing Hamilton's equations with a symplectic map). The round off acts as a noise whose amplitude is b^{-m} where b is the base and m the number of significant digits. In the Langevin equations describing the collisional effects we can neglect the round-off noise, whereas in the N body dynamics we cannot, since it makes the system irreversible. The information on any physical system being finite (position and velocity accuracy are limited by atomic size and thermal motion), irreversibily pops out naturally without any *ad hoc* hypothesis like Bolzmann or Prigogine did. As a consequence, computer simulation is closer to physical reality than the Platonic description provided by Hamilton's equations in a Euclidean phase space, where the specification of any state reequires infinite information.