Strongly Screened Vortex Lattice Model with Disorder

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The three dimensional XY model with quenched random disorder is studied in the strong screening limit, $\lambda \to 0$, by defect energy scaling at zero temperature. In zero external field we find that there exists a true superconducting phase with a stiffness exponent $\theta \simeq +1.0$ for weak disorder. For low magnetic field and weak disorder, we identify an ordered Bragg glass phase which is superconducting. For larger disorder or applied field, there is a non superconducting phase with $\theta \simeq -1.0$. We estimate the critical external field whose value is consistent with experiment.

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The phase diagram of type II superconductors in an external field has been the subject of intense theoretical and experimental investigation [1]. After the discovery of high-T_c materials, the role of both thermal and disorder induced fluctuations has been reconsidered, revealing many new interesting phenomena. In clean systems, it was realized [2] that, with increasing temperature, the Abrikosov lattice melts into an entangled vortex liquid via a thermally induced first-order transition. Experiments performed on thermodynamic quantities such as magnetization [3] and specific heat [4] confirmed the first order nature of the melting transition in YBCO and BSSCO materials. Yet more recent studies of the clean system in an external field [5] have shown that the low temperature vortex lattice melts at $T = T_M$ to a flux line liquid and at $T_L > T_M$ the lines become entangled and vortex loops proliferate.

At low and intermediate temperature, equilibration of vortex matter is very difficult due to the pinning of vortices by material disorder [6]. Pointlike disorder plays a major role and a glassy state results. The question of how quenched disorder affects the long range order of the vortex lattice has been controversial for some time. Although disorder was first argued to destroy the long range order of the Abrikosov lattice [7], more recent theories propose the existence of a novel phase of matter at low magnetic field, the so-called Bragg glass, which is almost as ordered as a perfect crystal [8]. Increasing the field effectively increases the disorder and the ordered Bragg glass transforms into a disordered phase characterized by vortex entanglement and the proliferation of dislocations [9]. The dramatic jump in the critical current associated with the melting of the Bragg glass [10] and the destruction of the Bragg peaks in neutron scattering [11] give experimental support to this scenario of a *field-driven* transition from order to disorder. However, the thermodynamic nature of the disordered phase is still controversial: pinned liquid [12] or vortex glass[13].

Two main theories have been proposed to describe the low-temperature glassy phase. The most recent approach [9] is based on the elastic theory of a vortex lattice in the presence of disorder. Within this approach it has been shown that disorder produces algebraic growth of displacements at short length scales and, at large scales, periodicity dominates resulting in a logarithmic correlation of displacements. An earlier alternative approach is the so-called *gauge glass model* which is the XY model with quenched random phase shifts with maximum disorder. In three dimensions, much evidence has accumulated that a true superconducting phase exists in the absence of screening, $\lambda = \infty$, from domain wall renormalization group analyses [14–16] and from Monte Carlo simulations [17]. In the low temperature phase, screening effects become important and, in the presence of screening, numerical simulations indicate that a thermodynamic ordered phase does *not* exist at finite temperature in this system [16, 18, 19]. This implies that finite screening of the vortex-vortex interaction is a relevant perturbation, but no definite conclusions could be reached due to the small system sizes studied. On the other hand, it has been shown recently [20] that the limiting case of vanishing screening length $\lambda \to 0$ can be analyzed using exact combinatorial methods for remarkably large system sizes. However, in contrast to a real superconductor in a magnetic field, the gauge glass is isotropic on average. Recent attempts to include the effect of anisotropy by introducing a uniform external field threading the system have failed [21, 22]. No finite temperature glass transition is found for any value of the field at large disorder. However, in the zero screening length limit the vortex-vortex interaction becomes isotropic [23] which implies that an isotropic gauge glass model may be a physically reasonable description.

In this Letter we identify unambiguously the transition from an ordered low-field Bragg glass phase to a disordered high-field phase at $T = 0$ by using defect energy scaling applied to the XY model with quenched random phase shifts with infinitely strong screening, $\lambda \to 0$. We show that the physically relevant limit is the case of *weak disorder* regime and we are able to take into account the periodicity of the ordered vortex lattice using periodic boundary conditions. We estimate the critical value of the external field as $h_c = \mathcal{O}(1)$ where $h = Ba_0^2/\Phi_0$ is the number of flux lines per plaquette. Here B is the actual field, a_0 is the lattice spacing and $\Phi_0 = 2 \times 10^{-7}$ Gauss$\cdot$cm^2 is the flux quantum. This value is compatible with the real magnetic field as measured experimentally [13]. In the strong screening limit, $a_0 \sim \lambda$, whose typical value in high-T_c superconductors is $\lambda \simeq 10^{-5}$ cm. This yields an estimate for the critical field $B_c \simeq 10^3$ Gauss. This is to be compared with the typical experimental value of $B_c \simeq 500$ Gauss in BSSCO [11].

We consider a three-dimensional XY model on a simple cubic lattice with quenched random phase shifts. In the vortex representation, the Hamiltonian is [24–26]

$$H = -\frac{1}{2}\sum_{i,j} G(i,j)(\boldsymbol{J}_i - \boldsymbol{b}_i)\cdot(\boldsymbol{J}_j - \boldsymbol{b}_j) \qquad (1)$$

We ignore boundary terms which, at least in the best twist approach, vanish by a proper choice of global twists [16]. The dynamical variables are the integer valued vorticities $\boldsymbol{J}_i$ on the links of the dual lattice and subject to the local constraint $(\nabla \cdot \boldsymbol{J})_i = 0$ at every site i. The $\boldsymbol{b}_i$ are quenched random fluxes on the dual lattice which are obtained from the circulation of the quenched vector potential $\boldsymbol{A}$ and by adding an uniform external field h in the $\hat{\boldsymbol{z}}$ direction

$$\boldsymbol{b}_i = \frac{1}{2\pi}[\nabla \times \boldsymbol{A}]_i + h\hat{\boldsymbol{z}} \qquad (2)$$

The vector potential $A_{\mu i}$ with $\mu = x, y, z$ is independently uniformly distributed $A_{\mu i} \in [0, 2\pi\alpha)$ with $0 \le \alpha \le 1$ and is defined on the bonds of the original lattice. The *disorder strength* α interpolates between two well known limits, the pure case ($\alpha = 0$), and the gauge glass ($\alpha = 1$). By construction, the fields $\boldsymbol{b}_i$ satisfy the divergenceless condition $(\nabla\cdot\boldsymbol{b})_i = 0$ on every site. $G(i,j)$ is the screened lattice Green's function

$$G(i,j) = \frac{(2\pi)^2}{L^3}\sum_{\mathbf{k}} \frac{1 - \exp[i\boldsymbol{k}\cdot(\boldsymbol{r}_i - \boldsymbol{r}_j)]}{2\sum_\mu(1-\cos k_\mu) + \lambda^{-2}} \qquad (3)$$

where $\boldsymbol{r}_i = (x_i, y_i, z_i)$ is the i-th site on the dual lattice and $k_\mu = 2\pi n_\mu/L$, with $\mu = x, y, z$ and $n_\mu = (1, \ldots, L)$.

In the strong screening limit, $\lambda \to 0$, the interaction of Eq. (3) becomes $G(i,j) = (2\pi\lambda)^2(1-\delta_{ij})$. Subtracting $(2\pi\lambda^2)$ and measuring energy in units of $(2\pi\lambda)^2$ yields the simple form

$$H = \frac{1}{2}\sum_i (\boldsymbol{J}_i - \boldsymbol{b}_i)^2 \qquad (4)$$

Note that there is no neutrality constraint when the interaction is screened with $\lambda < \infty$. This local form of Eq. (4) can be studied by very efficient combinatorial optimization algorithms [20, 28] on large systems. Any nonlocal terms such as vortex - vortex interactions or boundary terms [16, 18] in Eq. (1) render such algorithms useless.

To investigate whether a transition occurs, we use defect energy scaling. In this approach, one computes the energy $\Delta E(L)$ of a defect in a system of linear size L and fit to the *ansatz*

$$< \Delta E(L) > \sim L^\theta \qquad (5)$$

where θ is the stiffness exponent and $< \cdots >$ denotes an average over disorder. The sign of θ distinguishes two regimes: if θ is positive, inserting a defect costs an infinite energy in the thermodynamic limit and the system will be ordered at sufficiently small finite T. Conversely, if $\theta < 0$, large domains cost little energy and, at any $T > 0$, superconductivity will be destroyed. To calculate the defect energy we employ the method proposed by Kisker and Rieger [27], who restated the problem of finding the ground state for Hamiltonian (4) in terms of a minimum-cost-flow problem [20], where the cost functions are precisely given by $c_i(\boldsymbol{J}_i) = (\boldsymbol{J}_i - \boldsymbol{b}_i)^2/2$. This method makes use of the successive shortest path algorithm (SSPA) [28] to find the ground state configuration $\{\boldsymbol{J}^0\}$ for each realization of disorder. The global flux $\boldsymbol{f}$ associated with this configuration is given by

$$\boldsymbol{f} = \frac{1}{L}\sum_i \boldsymbol{J}_i^0 \qquad (6)$$

The elementary low energy excitation configuration $\{\boldsymbol{J}^1\}$ is obtained by gradually decreasing all costs in, say, the z direction until the global flux f_z jumps by one, $f_z \to f_z + 1$. The lowest energy excitation will be a global vortex loop encircling the $3D$ torus in the z direction. The defect energy is then obtained by $\Delta E = E(\{\boldsymbol{J}^1\}) - E(\{\boldsymbol{J}^0\})$. A more conventional way of determining the defect energy is to calculate the energy difference between periodic and antiperiodic boundary conditions, which amounts to adding a global twist of π along one spatial direction. In our case we can ignore the boundary terms because the global loop corresponds to a twist of 2π, which has no effect as the original Hamiltonian is invariant under a discrete gauge transformation modulo 2π. Remarkably large system sizes can be treated by applying the SSPA, while conventional methods such as repeated quenching or simulated annealing are much less efficient. In this work we study $L \times L \times L$ systems with $L \le 40$ for different values of α in the range $0 \le \alpha \le 1$ and magnetic field in the range $0 \le h \le 0.25$. The number of realizations of the random bonds varies from 500 for 40^3 systems up to 10^4 for the smallest ones. In principle, there is no upper limit on L except for time constraints.

Zero field: First, we investigate the effect of disorder strength α in the strongly screened model (4) in zero external field. We show the results for different values of α in Fig. (1). It is clear from Fig. (1) that there is a critical disorder strength $\alpha_c \simeq 0.5$ which distinguishes two different regimes: the pure case ($\alpha = 0$) where one simply has $< \Delta E(L) > \sim L^{+1}$ and the gauge glass case ($\alpha = 1$) where $< \Delta E(L) > \sim L^{-1}$. We note that finite size effects are quite strong in the narrow region around α_c.

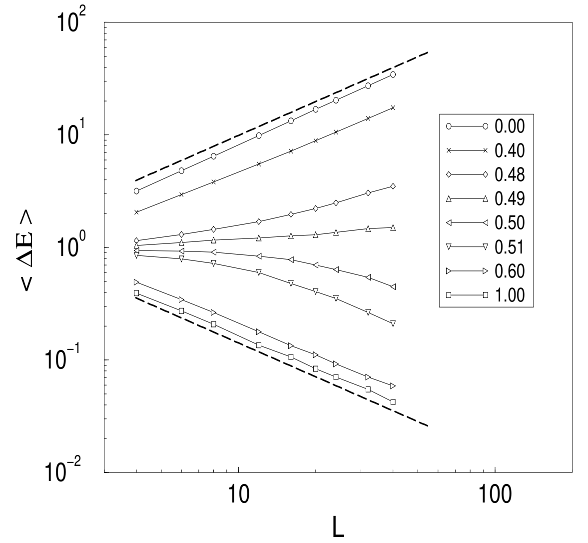

FIG. 1: Size L dependence of domain wall energy $\langle \Delta E \rangle$ in zero external field (log-log plot). The legend shows the magnitude of the disorder strength. Solid lines are guides for the eyes, dashed lines with slopes ± 1 are drawn for reference.

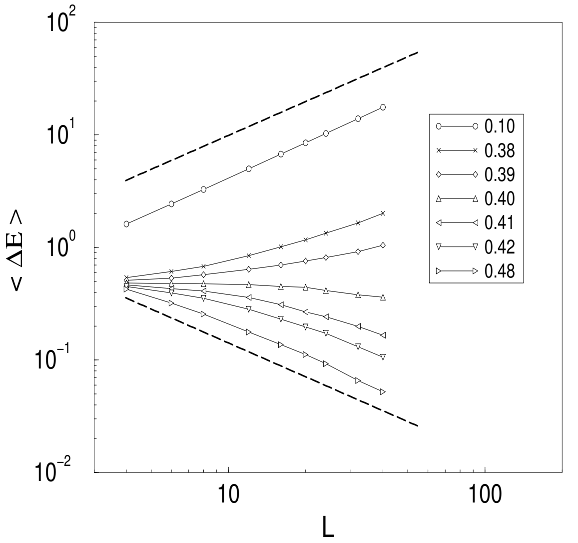

FIG. 2: L dependence of domain wall energy $\langle \Delta E \rangle$ for finite external field $h = 0.25$. Varying the disorder strength (see legend) changes the sign of the stiffness exponent θ. Dashed lines have slopes ± 1, while solid lines are guides for the eye.

However the tendency towards a transition with $T_c > 0$ to a superconducting ordered phase for $\alpha < \alpha_c$ and to a transition at $T_c = 0$ for $\alpha > \alpha_c$ seems quite clear. A value of $\alpha < \alpha_c$ is required for a true superconducting phase when the vortex - vortex interaction is screened. Previous studies [27] were not able to detect such a superconducting phase because they considered only the gauge glass case when $\alpha = 1 > \alpha_c$, but see [29].

Finite field: In the presence of weak disorder and applied field type II superconductors have fixed density of vortex lines which form a distorted Abrikosov lattice or Bragg glass [9] at low temperature. Increasing the field effectively increases the disorder and the Bragg glass phase is transformed into a disordered phase with no positional order of the vortex lattice. However, when $h \neq 0$, our model is not able to keep the matching between density of flux line and applied field because boundary terms are not included in Eq. (4). Moreover, we disregard interactions between vortex lines, which are at the origin of the Abrikosov lattice formation. Instead, we introduce an "effective" interaction by inserting a groove potential with the periodicity of the vortex lattice of $1/\sqrt{h}$. To calculate defect energy scaling it is important to keep periodic boundary condition in the system, so that methods which impose a fixed number of lines with fixed boundary conditions such as source/target [20] are not applicable. We have implemented the periodic groove potential as follows: We first add an external field h to *all* bonds in the z direction and calculate the ground state. In general the flux, Eq. (6), associated with this configuration is not equal to the flux implied by the external field, $h = Ba_0^2/\Phi_0$ as the $\lambda \to 0$ limit in Eq. (4) really describes a type I rather than a type II superconductor. To restore the correspondence, we introduce a potential Δ along the bonds *of the grooves* of the expected vortex lattice by $h \to h+\Delta$. We gradually decrease the costs of flux lines in the grooves by increasing Δ until the matching is satisfied with one flux line per groove. We thus obtain a ground state configuration with energy $E_0(L)$ with the necessary number $N = hL^2$ of lines. The procedure to find the defect energy is exactly the same as for zero field. Similar defect energy scaling for defect loops in the x direction transverse to the flux lines is also observed. Such a defect is induced by reducing the costs on all bonds in the x direction but, in a few samples, the number of lines along z changes as this is not fixed externally but is controlled by Δ. This must be kept at its value in the computation of the ground state in order to obtain an estimate of the defect energy. In Fig. (2) we show the behavior of the defect energy with system size L for a fixed value of external field $h = 1/4$ for different disorder strengths α. It is more convenient to fix the field and vary α because the period of the vortex lattice must be commensurate with the system size L which allows only restricted values of $h = NL^{-2}$ with N, L integers. At small disorder, $\alpha < 0.40$, we observe a positive stiffness exponent, which asymptotically tends to $\theta = 1$. Increasing the disorder, $\alpha > 0.40$, we find that the defect energy decreases with L and for large L, $< \Delta E(L) > \sim L^{-1}$. There is a critical value $\alpha_c(h = 0.25) \simeq 0.40$ separating an ordered from a disordered phase. Fig. (3) shows typical ground state configurations of the system. Below the

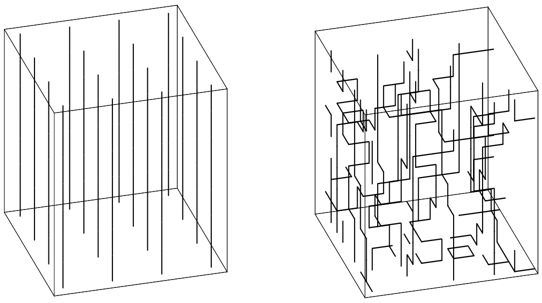

FIG. 3: Ground states for $h = 0.25$ in the ordered phase, $\alpha = 0.10$ (left) and in the disordered phase, $\alpha = 0.48$ (right).

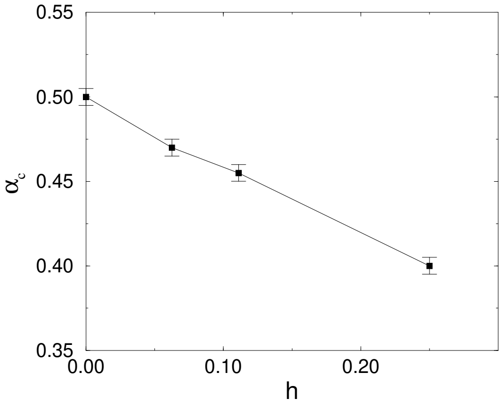

FIG. 4: Critical external field h_c versus disorder strength α.

critical disorder, Fig. (3) left, the lowest energy configuration forms an almost perfect vortex lattice, while above the critical disorder, Fig. (3) right, the lines are rough and entangled, as one would expect for a phase with proliferation of dislocations. We performed a similar analysis for different values of the field and find that $\alpha_c(h)$ decreases monotonically with increasing field as shown in Fig. (4).

In this Letter we have studied the strongly screened vortex glass in the presence of disorder and have successfully implemented a procedure to study the stability of order in the presence of an external field using periodic boundary conditions. It would be interesting to apply this method to probe directly the behavior of the model allowing for dislocations in the vortex lattice. This can be achieved by a shift by one vortex lattice spacing in the boundary conditions [30]. Work in this direction is in progress.

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