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Numerical study of random superconductors

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Abstract

The XY model with quenched random disorder is studied numerically at $T = 0$ by a defect scaling method as a model of a disordered superconductor. In 3D we find that, in the absence of screening, a vortex glass phase exists at low T for large disorder in 3D with stiffness exponent $\theta \approx +0.31$ and with finite screening and in 2D this phase does not exist. For weak disorder, a superconducting phase exists which we identify as a Bragg glass. In the presence of screened vortex–vortex interactions, the vortex glass does not exist but the Bragg glass does.

Keywords: Disorder; Superconductors; Vortex interactions; Bragg glass

1. Introduction

The phase diagram and superconducting properties of disordered superconductors have been of considerable interest for some time. The theoretical study of such systems is extremely difficult as the obvious approach is to find the low energy excitations about the ground state of a particular sample and compute the free energy $F(T,H,\{J\}) = -kT \ln Z(t,H,\{J\})$ where Z is the partition function for the particular realization of disorder $\{J\}$. Physical measurable quantities are appropriate derivatives of $\langle F \rangle$, the average over realizations of disorder. Various tricks such as the replica trick have been proposed to deal with quenched randomness.

We ask a very basic question about the existence of a low T phase by the method of finite size scaling of the defect energy. One argues that the disorder average of the energy $\langle \Delta E(L) \rangle$ of a disordering defect of size L scales as

$$\langle \Delta E(L) \rangle \sim L^{\theta} \quad (1)$$

for large L. Of course, this is not an experimentally measurable quantity nor does it give direct information about the putative phase transition at finite T. To obtain a theoretical prediction by e.g. Monte Carlo requires that the system is equilibrated which is almost impossible for a disordered system. A numerical estimate of the defect energy, $\Delta E(L)$, can be obtained by minimizing the energy, $E(L)$, of a 3D system of size L subject to appropriate boundary conditions. This avoids the necessity of thermal equilibration and can be done to sufficient accuracy to obtain very good fits to Eq. (1) and an estimate of the stiffness exponent θ. One argues that the system is ordered if $\theta > 0$ and disordered if $\theta < 0$ provided the defect generated by the boundary conditions is constructed properly [1].

2. Method and results

The model we study is an XY model with quenched random phase shifts with Hamiltonian

$$H = \sum_{\langle ij \rangle} V(\theta_i - \theta_j - A_{ij}) \quad (2)$$

where $V(\phi)$ is a 2π periodic function of ϕ. The sum is over all nearest neighbor pairs of sites and the coupling constants are assumed uniform $J_{ij} = J > 0$. The random bond variables A_{ij} are taken to be independent and uniform $A_{ij} \in (-\alpha\pi, +\alpha\pi]$ with $0 \leqslant \alpha \leqslant 1$. The parameter

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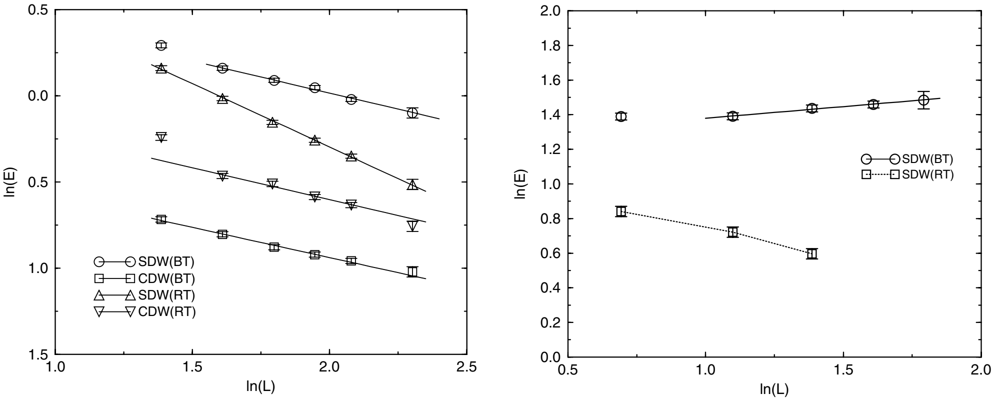

Fig. 1. Left: L dependence of ΔE_s^{BT}, ΔE_s^{RT}, ΔE_c^{RT}, ΔE_c^{BT} (top to bottom) for the 2D *XY* spin glass. Right: ΔE_s^{BT} and ΔE_s^{RT} for the 3D *XY* spin glass.

α is a measure of the strength of the disorder with $\alpha = 0$ no disorder and $\alpha = 1$ fully disordered. The next step is to rewrite the Hamiltonian of Eq. (2) in the dual vortex or Coulomb gas representation on a 3D torus to eliminate surface effects.

$$H = 4\pi^2 J \sum_{\mathbf{r},\mathbf{r}} \mathbf{p}_\mathbf{r} \cdot \mathbf{p}_{\mathbf{r}'} G(\mathbf{r} - \mathbf{r}') + \frac{\pi J}{2L^2}\mathbf{Q}^2$$
$$Q_x = \sum_{\mathbf{r}} \left(z p_\mathbf{r}^y - y p_{\mathbf{r}'}^z + L(z p_\mathbf{r}^y \delta_{y,1} - y p_\mathbf{r}^z \delta_{z,1}) \right) + 2L^2 p_{x1} \tag{3}$$

Here, $\mathbf{p}_\mathbf{r} = \mathbf{q}_\mathbf{r} - \mathbf{f}_\mathbf{r}$ where $\mathbf{q}_\mathbf{r}$ is the integer valued vector charge with $\nabla \cdot \mathbf{q}_\mathbf{r} = 0$ at each dual lattice site $\mathbf{r}$, $\mathbf{f}_\mathbf{r} = \nabla \times \mathbf{A}_\mathbf{r}$ is the random frustration at $\mathbf{r}$ and Q_y, Q_z are obtained by cyclic permutation of xyz in Q_x of Eq. (3). The mismatch or twist in the global phase shift around the torus in the $\hat{\mathbf{x}}$ direction is given by $f_{x1} = \frac{1}{2\pi}\sum_{x=1}^{L} A^x(x,1,1)$. The interaction is

$$G(\mathbf{r}) = \frac{1}{L^3}\sum_{\mathbf{k}\neq 0} \frac{e^{i\mathbf{k}\cdot\mathbf{r}} - 1}{6 - 2\sum_\mu \cos k_\mu + \lambda^{-2}} \tag{4}$$

where λ is the screening length with $G(r < \lambda) \sim r^{(2-D)}$ and $G(r > \lambda) \sim e^{-r/\lambda}$.

The absolute ground state energy $E_0(L)$ is obtained by minimizing the Hamiltonian $H(\mathbf{q}_\mathbf{r}, \mathbf{f}_\mathbf{r}, q_{x1}, f_{x1})$ with respect to the integer charges $\mathbf{q}_\mathbf{r}$, the integer global circulation q_{x1} *and* to the global frustrations $f_{\mu 1}$ which have the values $f_{\mu 1}^0$ at the global minimum for a given sample. This is analogous to periodic boundary conditions for a uniform ferromagnet. The minimization is done by simulated annealing [2] which works for an interacting system of Eq. (3). In 3D, we are limited to system sizes $L \leqslant 6$ by available computer power. In the infinite screening limit $\lambda \to 0$ there are some specialized algorithms which can find the ground state to machine accuracy in polynomial time [3] for large sizes L. The ground state energy is given by Eq. (3) with the appropriate values of the $\{\mathbf{q}_\mathbf{r}\}$ and the best twist f_{x1}^0. The energy of the system with a defect relative to the ground state $E_D(L)$ is obtained by minimizing H with respect to the $\{\mathbf{q}_\mathbf{r}\}$ with $f_{\mu 1} = f_{\mu 1}^0 + 1/2$ held fixed. The defect energy is $\Delta E(L) = E_D(L) - E_0(L) \geqslant 0$.

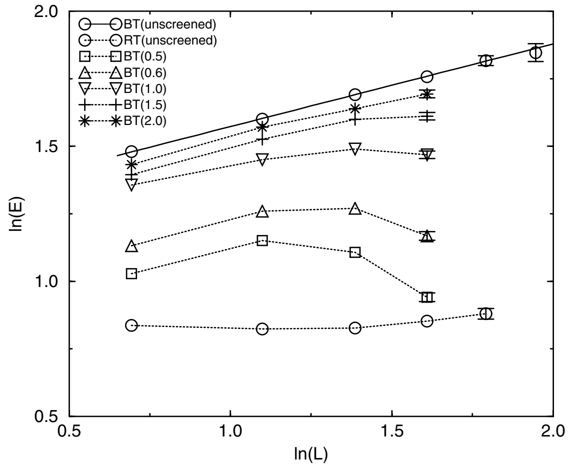

Fig. 2. L dependence of defect energy for the 3D gauge glass. Top curve is unscreened case. Bottom curve is a *RT* measurement for this. Other curves are for screened interactions with λ decreasing from top to bottom.

To check our method, we looked at the *XY* spin glass in 2D which has two stiffness exponents θ_s for the scaling of a phase defect and θ_c describing the scaling of a chiral defect. Since 2D is below the lower critical dimension we expect that $\theta_s = \theta_c < 0$. We find $\theta_s = \theta_c = -0.37 \pm 0.01$ which proves our algorithm [1]. Results are shown in Fig. 1. Encouraged by this, we investigated the *XY* spin glass and the *XY* gauge glass in 3D. The latter system is a zero field analogue of the putative vortex glass. We find $\theta_s = +0.10 \pm .02$ for the 3D spin glass and $\theta = +0.31 \pm .01$ for the gauge (vortex) glass (Fig. 2). These results imply that a vortex glass phase exists in 3D when

$\lambda = \infty$ but not if $\lambda < \infty$. A realistic superconductor has both disorder and $\lambda < \infty$. One could argue that $\lambda \to 0$ under scaling when

$$H = \frac{1}{2}\sum_{\mathbf{r}}(\mathbf{q}_{\mathbf{r}} - \mathbf{f}_{\mathbf{r}})^2 + O(\lambda^2) \tag{5}$$

We studied this with sizes $L \leqslant 40$ and find $\theta_s \approx -1.0$ for strong disorder and $\theta_s \approx +1.0$ for weak disorder which we postulate to be a Bragg glass [4]. This supports the idea that a highly disordered vortex glass does not exist in real systems and that a superconducting glass exists only as a weakly disordered Bragg glass.

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