First meeting of the (mostly)
YOUNG ITALIAN HYPERBOLICIANS
A workshop in Hyperbolic Dynamics
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Abstracts of the seminars

Claudio Bonanno (Università di Pisa)
Generalized transfer operators for the Farey map

Abstract. In this talk we study the properties of the transfer operators for the Farey map, which is an example of a map of the interval $[0, 1]$ expanding everywhere except for a neutral fixed point. We first recall some general results for maps of this kind and then show some recent results for the Farey map.

Francesco Cellarosi (Princeton University)
Limiting curlicue measures for theta sums

Abstract. I will discuss the ensemble of curves generated by the classical theta sums $\sum_{n=0}^{N-1} e^{\pi i n^2 \alpha}$, and prove that they are distributed according to some limiting probability measure as $N \to \infty$. Despite the potentially intimidating number-theoretical origin of theta sums, the proof is purely dynamical. I will explain the connection between the geometric multi-scale structure of the curves and the continued fraction expansion of α with even partial quotients, whose associated Gauss-like map preserves an infinite ergodic measure. The main tools in the proof are an acceleration of this map, a renormalization procedure and a renewal-type limit theorem for the denominator of the above continued fractions. The result generalizes theorems by Marklof and Jurkat and van Horne.

Giampaolo Cristadoro (Università di Bologna)
Statistical properties of intermittent maps with unbounded derivative

Abstract. We study the ergodic and statistical properties of a class of maps of the circle and of the interval of Lorenz type which present indifferent fixed points and points with unbounded derivative. These maps have been previously investigated in the physics literature. We prove in particular that correlations decay polynomially, and that suitable Limit Theorems (convergence to Stable Laws or Central Limit Theorem) hold for Hölder continuous observables. We moreover show that the return and hitting times are in the limit exponentially distributed. (Joint work with N. Haydn, P. Marie and S. Vaienti.)

Gianluigi Del Magno (Universidade Técnica de Lisboa)
Filtering for chaotic maps

Abstract. The 'Filtering problem' consists of estimating the current state of a dynamical system from a record of noisy measurements. More precisely, consider two stochastic processes $\{X_n\}$ and $\{Y_n\}$ with $\{Y_n\}$ being a random perturbation of $\{X_n\}$. We consider $\{X_n\}$ as an unobservable process and $\{Y_n\}$ as a measurement process. The 'Filtering problem' then consists of computing the conditional expectation of X_n given the observations $Y_1, \ldots, Y_n$. The asymptotic properties of the filtering process $\{Z_n\}$ are of great interest both from a theoretical perspective as well as in applications. In this talk I will report on some preliminary results (joint work with Jochen Bröcker) concerning the asymptotic properties of the filtering process when $\{X_n\}$ is generated by the iterations of an expanding map.

Jacopo De Simoi (University of Maryland)

Standard pairs in an anti-integrable limit of the standard map

Abstract. Starting from a simple Hamiltonian system modeling Fermi acceleration, it is natural to study a one-parameter family of twist maps on the cylinder that can be viewed as anti-integrable limits of the standard map. We conjecture that for all maps of such family, orbits which undergo Fermi acceleration, although being a full Hausdorff dimension set, have measure zero. In this seminar we will support the conjecture by showing some results obtained using standard pairs techniques developed by D. Dolgopyat.

Stefano Galatolo (Università di Pisa)

Shrinking targets, decay of correlations and arithmetical properties

Abstract. We will consider the time which is needed for a typical point to enter a sequence of decreasing balls. In many systems the typical time increases as the inverse of the measure of the balls. More precisely let $\tau_r(x, x_0)$ be the time needed for a point x to enter for the first time a ball $B_r(x_0)$ centered in x_0, with small radius r. We consider $\liminf_{r\to 0} \frac{\log \tau_r(x,x_0)}{-\log r}$. In systems with generic arithmetical properties, or fast decay of correlations, the above limit is related to the local dimension $d_\mu(x_0)$ of the invariant measure at x_0, but there are systems for which this relation does not hold at all. We will survey positive and negative results on this question, relating them to some other topics in the literature, as dynamical Borel Cantelli results and Logarithm laws. In particular we will show a class of mixing examples, constructed by reparametrizing suitable flows for which the above limit goes to infinity (while the dimension is 3), and some recent results about the Lorenz system.

Paolo Giulietti (La Sapienza Università di Roma)

Anosov flows and zeta functions

Abstract. We present work in progress (joint with C. Liverani and M. Pollicot) on the study of the Ruelle zeta function on a manifold of variable negative curvature: we claim that for any C^∞ Anosov flow the Ruelle zeta function is meromorphic on the entire complex plane. We explore the possibility of extending our results to a special family of L-functions. Note that our techniques deal directly with the transfer operator rather then resorting to cumbersome Markov partitions.

Marco Lenci (Università di Bologna)

On infinite-volume mixing

Abstract. In the context of the long-standing issue of mixing in infinite ergodic theory, we introduce the idea of infinite-volume mixing for dynamical systems preserving a nonnormalizable measure. The idea is borrowed from statistical mechanics and appears to be very relevant, at least for extended systems with a direct physical interpretation. We discuss the pros and cons of a few mathematical definitions that can be devised, exemplifying them by means of a prototypical class of infinite-measure preserving dynamical systems, namely, the random walks.

Maria José Pacifico (Universidade Federal do Rio de Janeiro)

Mixing rate for semi-dispersing billiards with non-compact cusps

Abstract. Since the seminal work of Sinai people have studied the properties of planar billiards. Among them is the decay of correlations. There are many results in this direction for billiards in spaces with finite measure. However, it seems that until now nothing was known about mixing and the rate of mixing in the case of infinite measure, which is the case of billiards with tables presenting a non-compact cusp. The first problem is to find a "good" definition of mixing for this case. We shall prove that with the definition given by Krengel and Sucheston [1], the kind of billiards studied by Lenci [2] are mixing and that the speed of mixing is polynomial.

[1] U. Krengel and L. Sucheston, *"On mixing in infinite measure spaces"*, Z. Wahrscheinlichkeitstheorie und Verw. Gebiete **13** (1969), 150–164

[2] M. Lenci, *"Semi-dispersing billiards with an infinite cusp. I"*, Comm. Math. Phys. **230** (2002), no. 1, 133–180

Christoph Schumacher (Universität Erlangen-Nürnberg)

A geometric Markov partition for a random potential

Abstract. In this talk I present a class of hyperbolic random dynamical systems and show how to construct a geometric Markov partition *à la* Series [3].

[3] C. Series, *"Geometrical Markov coding of geodesics on surfaces of constant negative curvature"*, Ergodic Theory Dynam. Systems **6** (1986), no. 4, 601–625

Marcello Seri (Università di Bologna)

Recurrence for quenched random Lorentz tubes

Abstract. We define a class of quenched random Lorentz gases, infinitely extended only in one dimension, for which almost sure recurrence can be proved. We use two main tools: first we reduce the extended system to a proper finite-measure system and then we prove its ergodicity. Applying a powerful theorem by Schmidt to this result one has the proof. Finally, we discuss some possible generalizations. (Joint work with G. Cristadoro and M. Lenci.)