$x^2+y^2+10z^2=-1$ rotazione attorno a una retta

$x^2+y^2+z^2=-1$ " " " un punto

$x^2+5y^2+10z^2=-1$ non di rotazione

Trovare le eventuali quadriche di rotazione della famiglia

$$11x^2+(\gamma+1)y^2+11z^2+2\gamma y-18xz=0$$

e i loro spazi di rotazione.

$$A_\gamma=\begin{pmatrix} 0 & 0 & \gamma & 0 \\ 0 & 11 & 0 & -9 \\ \gamma & 0 & (\gamma+1) & 0 \\ 0 & -9 & 0 & 11 \end{pmatrix}$$

$$\begin{vmatrix} (\lambda-11) & 0 & 9 \\ 0 & (\lambda-\gamma-1) & 0 \\ 9 & 0 & (\lambda-11) \end{vmatrix} = (\lambda-\gamma-1)\begin{vmatrix} (\lambda-11) & 9 \\ 9 & (\lambda-11) \end{vmatrix} = (\lambda-\gamma-1)(\lambda-11+9)(\lambda-11-9) =$$

$$=(\lambda-\gamma-1)(\lambda-2)(\lambda-20)$$

Autovalori: $\lambda_1=\gamma+1$ $\lambda_2=2$ $\lambda_3=20$

rotazione $\Leftrightarrow \lambda_1=\lambda_2 \vee \lambda_1=\lambda_3$

$\lambda_1=\lambda_2 \Leftrightarrow$
$\gamma+1=2 \Leftrightarrow \gamma=1$

$\lambda_1=\lambda_3 \Leftrightarrow$
$\gamma+1=20 \Leftrightarrow \gamma=19$

$$A_\gamma = \begin{pmatrix} 0 & 0 & \gamma & 0 \\ 0 & 11 & 0 & -9 \\ \gamma & 0 & (\gamma+1) & 0 \\ 0 & -9 & 0 & 11 \end{pmatrix}$$

$\gamma = 1$ $\quad A_1 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 11 & 0 & -9 \\ 1 & 0 & 2 & 0 \\ 0 & -9 & 0 & 11 \end{pmatrix}$

$\lambda = 2$ $\quad \begin{pmatrix} -9 & 0 & +9 \\ \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{+9} & \cancel{0} & \cancel{-9} \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} 0 \\ \cancel{0} \\ \cancel{0} \end{pmatrix}$ $\quad U_2 : -9l + 9n = 0$

$\mathcal{B}_{U_2} : ((1,0,1), (0,1,0))$

$(0\,1\,0\,1)\, A_1 \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$ $\quad 2x + 2z = 0$

$(0\,0\,1\,0)$ asse dir $\quad 1 + 2y = 0$

NB: ogni $\alpha(1,0,1) + \beta(0,1,0)$ è autovettore, perciò ogni $\alpha(2x+2z) + \beta(1+2y) = 0$ è un piano principale

Qual è l'altro piano principale?

$\lambda = 20$ $\quad \begin{pmatrix} 9 & 0 & 9 \\ 0 & 18 & 0 \\ \cancel{9} & \cancel{0} & \cancel{9} \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \cancel{0} \end{pmatrix}$ $\quad U_{20} : \begin{cases} 9l + 9n = 0 \\ 18m = 0 \end{cases}$

$\mathcal{B}_{U_{20}} = ((1,0,-1))$

$(0\,1\,0\,-1) \cdot A_1 \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0$ $\quad 20x - 20z = 0$

$$A_\gamma = \begin{pmatrix} 0 & 0 & \gamma & 0 \\ 0 & 11 & 0 & -9 \\ \gamma & 0 & (\gamma+1) & 0 \\ 0 & -9 & 0 & 11 \end{pmatrix}$$

$\gamma = 19$

$$A_{19} = \begin{pmatrix} 0 & 0 & 19 & 0 \\ 0 & 11 & 0 & -9 \\ 19 & 0 & 20 & 0 \\ 0 & -9 & 0 & 11 \end{pmatrix}$$

$$\lambda = 20 \quad \begin{pmatrix} 9 & 0 & 9 \\ \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{9} & \cancel{0} & \cancel{9} \end{pmatrix} \begin{pmatrix} l \\ m \\ n \end{pmatrix} = \begin{pmatrix} 0 \\ \cancel{0} \\ \cancel{0} \end{pmatrix}$$

$V_{20}: \; 9l + 9n = 0$

$\mathcal{B}_{V_{20}} = \big((1,0,-1),(0,1,0)\big)$

$$\begin{matrix} (0\ 1\ 0\ -1) \\ (0\ 0\ 1\ 0) \end{matrix} \; A_{19} \cdot \begin{pmatrix} 1 \\ x \\ y \\ z \end{pmatrix} = 0 \quad \begin{cases} 20x - 20z = 0 \\ 19 + 20y = 0 \end{cases}$$

asse di rot.

$$\begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 2 & \\ & & & 3 \end{pmatrix} \quad \begin{pmatrix} -1 & & & \\ & 2 & & \\ & & 2 & \\ & & & 3 \end{pmatrix} \quad \begin{pmatrix} -1 & & & \\ & 2 & & \\ & & 2 & \\ & & & 2 \end{pmatrix}$$

$$\begin{pmatrix} -1 & & & \\ & -1 & & \\ & & 2 & \\ & & & 3 \end{pmatrix} \quad \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & 2 & \\ & & & 2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 2 & \\ & & & 3 \end{pmatrix} \quad \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 2 & \\ & & & 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & & \\ 0 & & 2 & \\ 1 & & & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 2 & & \\ 0 & & 2 & \\ 1 & & & 0 \end{pmatrix} \quad \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & & \\ 0 & & -2 & \\ 1 & & & 0 \end{pmatrix}$$

$$a_{00}X_0^2 + 2a_{01}X_0X_1 + 2a_{02}X_0X_2 + a_{11}X_1^2 + 2a_{12}X_1X_2 + a_{22}X_2^2 = 0$$

$$(a_{00}, a_{01}, a_{02}, a_{11}, a_{12}, a_{22})$$

$$\overline{P}$$

$$\begin{cases} a_{00}\overline{X}_0^2 + 2a_{01}\overline{X}_0\overline{X}_1 + \cdots = 0 & \overline{P} \\ a_{00}\overline{\overline{X}}_0^2 + 2a_{01}\overline{\overline{X}}_0\overline{\overline{X}}_1 + \cdots = 0 & \overline{\overline{P}} \\ a_{00}\tilde{X}_0^2 \cdots = 0 & \tilde{P} \\ a_{00}\tilde{\tilde{X}}_0^2 \cdots = 0 & \tilde{\tilde{P}} \\ a_{00}X_0^{*2} + \cdots = 0 & P^* \end{cases}$$