KP^4

RP^5

$\beta_5 = 1 - 0 = 1 \quad H_5 \cong \mathbb{Z}$

$\beta_4 = 1 - 1 = 0 \quad H_4 \cong 0$

$E^4 = (0) \quad \beta_4 = 1 - 0 - 1 = 0 \quad H_4 \cong 0$

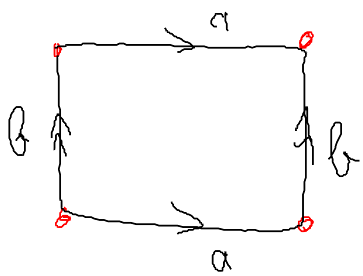
$E^3 = (2) \quad \beta_3 = 1 - 1 - 0 = 0 \quad H_3 \cong \mathbb{Z}_2$

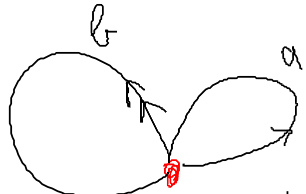
$E^2 = (0) \quad \beta_2 = 1 - 0 - 1 = 0 \quad H_2 \cong 0$

$E^1 = (2) \quad \beta_1 = 1 - 1 - 0 = 0 \quad H_1 \cong \mathbb{Z}_2$

$E^0 = (0) \quad \beta_0 = 1 - 0 = 1 \quad H_0 \cong \mathbb{Z}$

$\langle a \rangle$





$\langle a, b \mid a b a^{-1} b^{-1} \rangle$

$$\begin{tikzcd}
A \times B \arrow[r, "\psi"] \arrow[dr, "\varphi"', "\text{bilineare}"'] & A \otimes B \arrow[d, "f", "\text{lineare}"'] \\
& C
\end{tikzcd}$$

$$(a,b) \mapsto a \otimes b$$

$\mathbb{Z} \otimes \mathbb{Z}$

generato da tutti gli $m \otimes n$

$m \otimes n = n(m \otimes 1) = mn(1 \otimes 1)$

$\mathbb{Z} \otimes \mathbb{Z}$ è lo $\mathbb{Z}$-modulo generato da $1 \otimes 1$.

perciò $\mathbb{Z} \otimes \mathbb{Z} \cong \mathbb{Z}$

$\mathbb{Z} \otimes \mathbb{Z}_3$

generato dagli $m \otimes \tilde{n}$ $[n]_3$

$m \otimes \tilde{n} = m(1 \otimes \tilde{n}) = (1 \otimes \widetilde{(m \cdot n)})$

quindi ho solo 3 elementi: $1 \otimes \tilde{0}, 1 \otimes \tilde{1}, 1 \otimes \tilde{2}$

$\mathbb{Z} \otimes \mathbb{Z}_3 \cong \mathbb{Z}_3$

$[m]_2$

$\mathbb{Z}_2 \otimes \mathbb{Z}_3$

generato dagli $\overline{m} \otimes \tilde{n}$

$$\overline{m} \otimes \tilde{n} = mn(\overline{1} \otimes \tilde{1})$$

se $m \cdot n$ è pari allora $mn(\overline{1} \otimes \tilde{1}) = (\overline{mn}, \tilde{1}) =$

$$= (\overline{0}, \tilde{1}) = (\overline{0}, \tilde{0})$$

se $m \cdot n$ è dispari $mn(\overline{1} \otimes \tilde{1})$ è

è banale

- $\overline{1} \otimes \tilde{0} = \overline{0} \otimes \tilde{0}$
- $\overline{1} \otimes \tilde{1} = \overline{1} \otimes \tilde{4} = \overline{4 \cdot 1} \otimes \hat{1} = \overline{0} \otimes \hat{1} = \overline{0} \otimes \tilde{0}$
- $\overline{1} \otimes \tilde{2} = \overline{2 \cdot 1} \otimes \hat{1} = \overline{0} \otimes \hat{1} = \overline{0} \otimes \tilde{0}$

$$\mathbb{Z}_p \otimes \mathbb{Z}_q \cong \mathbb{Z}_{MCD(p,q)}$$