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Opponent report

TEACHERS' CONVICTIONS ON MATHEMATICAL INFINITY

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The thesis is divided in 4 chapters and it faces the conceptions of the taught of the concept of infinity.

The first chapter develops a punctual analysis of the history of the concept of infinity from the ancient history to our days. The sense of this historical way is that to track the historical conceptions down is from the semantic point of view that syntactic. In both the cases this analysis is a fundamental pre-requisite to be able to study and to analyze the conceptions of the teachers through the different linguistic registers.

The work is conducted with a lot of scrupulousness and scientific correctness.

At the end of the chapter I would have added a summarized conceptual map of the historical ways.

The second chapter faces the international researches context and the description of theoretical framework.

The work is developed and with opportune scientific references. The historical way of the obstacles, of the misconceptions and of the mental images.

Every of these models of the search in didactics they are inserted in different paradigms. Silvia Sbaragli succeeds in organizing a theoretical framework of reference that keeps in mind of these different paradigms. This aspect is analyzed as she affirms from the "*point of view of fundamental didactics*".

Elements characterising this side are:

- *various learning theories;*
- *role and nature of conceptions;*
- *epistemological obstacles theory.*

The third chapter represents the heart of the work of thesis. The analysis of the conceptions of the teachers is well harmonized with the study historical-epistemological of the chapter 1 and with the analyses of fundamenta of search in didactics of the chapter 2.

The hypotheses of research are express in clear and well tied up form to the historical reflections of the chapter 1.

Ample well connected bibliography with the experimental reasonings.

H.1 We believe that mathematical infinity is a rather unfamiliar subject for most primary school teachers, both from an epistemological and from a cognitive point of view. We therefore thought that teachers would not be able to handle infinity and to conceive it as a mathematical object. Consequently, we assumed that teachers would stick to naive convictions as for example: infinity is *nothing but indefinite, or infinity is synonymous with unlimited, or else infinity is a very large finite number [convictions that were present over the centuries throughout the history of this topic, see chapter 1, in particular they are to be traced back in the statements of Nicholas of Cusa (1400 or 1401-1464)].*

H.2 We believe that primary schoolteachers normally provide pupils with intuitive models of mathematical infinity, starting from the early years of primary school.

Moreover, if teachers' naive convictions, assumed in H.1, were verified, they would condition (in our opinion) the models provided to pupils. We assumed that teachers provided intuitive models that they considered correct, but in fact they were based on misconceptions. In order to verify this hypothesis, we judged that it would be interesting to analyse accurately the teachers' statements and their way of expressing ideas.

H.3 We assumed that, if the two above-mentioned hypotheses had been proved true, beside epistemological obstacles that the study of mathematical history and the criticism of its fundamentals have highlighted, we would have been able to trace obstacles of didactical nature too. One of the obstacles we thought we would encounter is bound to a naive idea of infinity as a synonym for unlimited, a conviction which is in contrast with the concept of the infinity of points in a segment, a segment being limited though constituted of infinite points. One more obstacle we thought we would find is bound to the idea of infinity, considered as a large natural number [see ch. 1: Anaximander of Miletus (610 B.C. – 547 B.C.) and Nicholas of Cusa (1400 or 1401-1464)], it follows that the same procedures applied to finite sets are automatically transferred to infinite sets, which are seen as very large finite sets. Another didactical obstacle, often highlighted by Arrigo and D'Amore (1999, 2002), that we were confident we would come across, is the "model of the necklace" as the two authors call it. Students often point it out as a suitable model to visualise the points on a straight line, and they indicate their primary school teachers as the source of this model that withstands all subsequent attacks. (Arrigo and D'Amore, 1999; 2002) Our hypothesis was therefore that we would encounter didactical obstacles, deriving from typical models, usually introduced by primary school teachers.

The falsification of the hypotheses is tightly tied to analysis type qualitative is in the experiences with the teachers (chapter 3) both with the students (chapter 4).

The point of view of the student has put in evidence in the fourth chapter. Numerous experiences of teaching/learning are introduced of the concept of infinity. These experiences are been analyzed in course of formation of teachers. Experiences are been translated then in articles published from Sbaragli in Journals. For example, "Meetings with mathematics to Castel San Pietro", to a seminar for primary and lower secondary school teachers called: *"Infinities and infinitesimals in primary and lower secondary school"*.

A conclusion of the thesis concerns the didactic transposition. You could hypothesize then that the didactic transposition in class is not anything else other than a passage from the intuitive phase of the concept, consolidated in the time in the mind of the teacher, to the intuitive phase of the student; that it comes so, accordingly confirmed, and strengthened. This attitude is source of didactic obstacles that prevents the understanding of the mathematician concept of infinity.
P1. Is there awareness among the teachers on what he intends for endless mathematician and on as it works in the epistemology and in the knowledge?
(C'è consapevolezza tra gli insegnanti per quello che si intende con infinito matematico anche dal punto di vista epistemologico e della conoscenza?)
P2. Do the teachers furnish intuitive models to his/her own students on this matter since the first years of elementary school?
(Forniscono gli insegnanti dei modelli intuitivi relativi a questo argomento ai propri studenti già dai primi anni di scuola elementare?)
P3. Can The convictions of the teachers be cause of didactic obstacles that feeds the epistemological obstacles already underlined by the international search?

(Possono le convinzioni degli insegnanti essere causa di ostacoli didattici che a loro volta alimentano gli ostacoli epistemologici già evidenziati dalla ricerca a livello internazionale?)

To these questions she has tried to answer the thesis of Silvia Sbaragli. (A queste domande la candidata ha provato coscienziosamente e con serietà a fornire delle risposte.) Develops with seriousness and rigor. Qualitative analyses are suitable to the falsification of the hypotheses.

Therefore I recommend ***to accept the Thesis and after its successful defence to grant the applicant the academic degree PhD.***

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