

## Report on doctoral thesis

### *Teachers Convictions on Mathematical Infinity*

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Given work is as doctoral thesis relatively comprehensive – 178 pages. Work consists of introduction and four chapters. Complexity of approach to problem is possible to be seen in volume of used literature – 145 publications, the most of them from the last decade of the 20<sup>th</sup> and from the 21<sup>st</sup> century. It shows actuality of theme. Deepness of treating is also visible in total time of this research; it was more like seven years.

The 1<sup>st</sup> chapter describes transparently historical development of conception of mathematical infinity. It starts with antique and ends with consequences of Hypothesis of continuum and Pean's inductive axiom. Single stages of development are written critically with rich literal references. It needed to emphasize that this approach doesn't lower survey. On the contrary – it gives possibility thoroughly acquaint with epistemological obstacles, which there were at development of conception of mathematical infinity. The volume of chapter – 52 pages – can be taken as integrated vocational materials.

The 2<sup>nd</sup> chapter – the shortest – contains description of key words of actual didactic of mathematics (didactical contract, visions, models, conflicts, incorrect opinions, obstacles). This chapter describes in more details also „didactical triangle“. This part thus produces theoretical base of problematic stated in this thesis.

The 3<sup>rd</sup> chapter comes out from description of theoretical frame of research about ideas of mathematical infinity between teachers of primary schools and lower secondary schools. Author formulates problem and basic hypothesis of research (p. 70 – 72) and she describes methodology of research. The next part shows results of questionnaires (test with 13

questions) and gives main results with demonstrations of typical reactions of teachers to questions. Interesting is result that practically all teachers have ideas of mathematical infinity on the level of intuitive conception. It was found out that epistemological and cognitive meaning of notion „mathematical infinity“ is for teachers de facto unknown, mostly is understood like opposite of notion „finiteness“. Comprehensive is explanation of reasons to use phenomenon „flattening“ and „dependence“ described in theoretical frame of research. Research has clearly shown that important positions are taken with didactical obstacles, for which is not given enough care as for epistemological obstacles. This discovery shows the need to rebuilt teachers training of mathematics, also confirms possibility to change the content of teaching the mathematics on the primary schools and lower secondary schools. This part of the thesis was publicized in other author's work.

The 4<sup>th</sup> chapter describes innovational course of mathematical teacher's training, which was realized with aim to improve apprehension of notion „infinity“, from the results it comes out to attention of experiment with pupils. This experiment was orientated to view of infinity also another basic geometrical notions (point, line, plane, space, etc). With using methodology TEP was realized analyze of results obtained from pre-schools children to students which leave secondary schools. Stated results have wide reach to other mathematical objects as infinity and very validly confirm relations between mathematical notions and mathematical objects, demonstrated with reflections about Duvall's paradox. Single parts are properly add up with demonstrations of TEP and of answers of teachers. In the end of chapter are given some proposals of other activities, which can eliminated found groins of mathematical representation of students and better understanding of notion infinity of teachers. In very end of the chapter author engages actual international research orientated to „sense of infinity“. Results will be publicized alone and aren't in this work.

Given thesis is „work of art“. Despite of voluminosity is work written clearly and transparently. Attention is given to contents and also to structure of work, which is comprehensive. Thanks to precise language style is work attractive for reader. It is also thanks to fact that deeper didactical reflections are added up ex una parte with lot of references to studied and used literature, on the other side with results of own researches. Briefly said – work documents high level of knowledge of problematic and ability of own creative academic work by the way which is evidently original contribution to development of didactic of mathematics. For me is pleasant how comprehensive and inspiring whole text is despite of my

fractional knowledge of French school of didactic. I was also amazed that author comes out from the works of her adviser (together 29 quoted works). Maybe only one unanswered question after reading this work for me remains question about generality of formulated results. I think whether used approach would reach the same results also in the case if in the center of observation is less complicated mathematical notion as is infinity.

Given work with its form highly exceed requirements set to PhD's thesis. For this fact I recommend this work to be regarded and I suggest after successful advocacy accord to Silvia Sbaragli title **philosophiae doctor** in the specialization 11-17-9 Theory of teaching mathematics.

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opponent