Soybean Inventory and Forward Curve Dynamics

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We present two results concerning soybean prices. First, we exhibit a simple relationship between stocks and price volatility. The observation of an increasing price volatility with decreasing inventory is often mentioned in the literature, but has so far been documented using a proxy for inventory (see Fama and French 1987, 1988; Litzenberger and Rabinowitz 1995). Instead, we reconstruct a yearly, quarterly, and monthly database of worldwide soybean inventories using aggregate data from the United States, Brazil, and Argentina. We show that under all time scales, price volatility is an increasing linear function of inverse inventory, which we term “scarcity.” Second, we show how the addition of the factor scarcity in a state-variable approach to the dynamics of the term structure of soybean forward prices improves the quality of the fit. We document this property on a 25-year database of CBOT futures contracts and show that the superior accuracy also affects long-maturity futures contracts, an important property for the valuation of long-term origination contracts between producing countries and the agrifood industry.

Key words: term structure of soybean forward prices; state variables; scarcity-related volatility

History: Accepted by Ravi Jagannathan, finance; received January 30, 2003. This paper was with the authors 6 months for 2 revisions.

1. Introduction
One of the characteristics of agricultural commodities traded in spot or futures markets is the high volatility of prices. Deaton and Laroque (1992), using annual price data on 12 agricultural commodities from 1900 to 1987, find notably high coefficients of variation (e.g., 0.6 for sugar). Most series of agricultural spot prices show occasional sharp rises. Kaldor’s (1939) explanation goes as follows: “In the case of agricultural crops, the supply curve is much less elastic and is subject to frequent and unpredictable shifts, due to the weather. It is impossible to foretell the size of the crop a year ahead, and it is not possible to say when prices will revert to normal.” Two additional factors may be the deterioration of agricultural products making storage more expensive relative to other types of commodities such as metals (Fama and French 1987), and the fact that some of them are not continuously produced (Scheinkman and Schechtman 1983). Our goal in this paper is to show the importance of inventory in explaining soybean price volatility and forward curve dynamics.

In models of price determination for storable commodities, knowledge of the quantities produced and stored is obviously important in the derivation of testable predictions about the trajectory of prices. Geman and Vasicek (2001) analyze the valuation of forward and futures prices of electricity and contrast it with the case of storable commodities. On a related matter, inventories have been analyzed in terms of convenience yield. The “theory of storage” of Kaldor (1939), Working (1948, 1949), Brennan (1958), and Telser (1958) explains convenience yields as an embedded timing option. For example, there may be a convenience yield from holding inventories of such commodities such as soybean or wheat because they are inputs to the production of other commodities (e.g., soya meal or flour). Or, there may be a convenience yield because the existence of stocks allows one to meet unexpected demand, and avoid the nuisance and cost of revisions of the production schedule.

Agricultural commodities also differ from stocks, bonds, and other financial assets because of the pronounced seasonals in both price levels and volatilities. Spot prices for agricultural commodities usually rise between harvests and fall during harvests. Seasonality and periodicity in agricultural commodity prices and inventories are due to seasonal cycles of production (see Brennan 1959, Fama and French 1987), but also to other exogenous factors such as climatic conditions (Chambers and Bailey 1996). The economy is different at different dates because of existing inventory levels and current and expected future demand (e.g., December is different from May in terms of decisions relative to soybean storage). Our first contribution in this paper is to empirically document that in the case of the soybean, price volatility is an increasing function of inverse inventory level; we term this quantity scarcity. In this order, we reconstruct a 25-year database of soybean world
inventories and run a regression of the spot price volatility on the scarcity variable. A positive correlation is observed not only at the annual timescale, but also on subannual periods (after a deseasonalization procedure meant to give significance to the results). As a side note, we also investigate the extent to which the U.S. soybean inventory is representative of the world inventory and show that U.S. stocks have almost the same impact on volatility as world stocks do.

We then propose a state-variable approach to the dynamics of the term structure of soybean forward prices, successively using two and three factors: The first two factors are chosen to be the spot price and the short-term mean of its nonseasonal component. Seasonalities in spot price and volatility are controlled for by appropriate deterministic functions. Consistently with the above analysis, we then add scarcity as a third state variable. We study a 25-year database of soybean futures traded on the CBOT to compare the quality of the two models: Using a subset of the data to estimate the models, we then test them out of sample and exhibit a clear superiority of the three-factor model. Let us note that the use of the spot price as a first state variable in the modeling of forward curve dynamics is a standard one, as parallel moves in forward curves are mainly explained by changes in the spot price.1 We choose the short-term mean of the stochastic component of the spot price as a second state variable because it may be viewed as its value under “normal” supply-demand conditions and plays a key role in explaining commodity prices (besides inventories, other factors such as agricultural policies or the prospect of the next crop size may affect the short-term mean of the stochastic component).

The addition of scarcity as a third state variable is motivated by our empirical findings on the role of inventory in explaining spot prices and spot price volatility and, in turn, the forward curve. We believe our model is general enough to be applied to other agricultural commodity markets or energy commodities as long as worldwide stocks may be thoroughly identified.

The remainder of the paper is organized as follows. Section 2 provides additional references pertinent to the subject of our study. In §3, we motivate our approach by empirically exhibiting the link between scarcity and spot price volatility, the result holding under different timescales. The two- and three-state variable models are described, respectively, in §§4 and 5. The results of the estimation of the two models and the quality of the fitting obtained in sample and out of sample are presented in §6. Section 7 contains concluding comments.

1 A principal component analysis of soybean forward curve changes over a 25-year time period is available from the authors.

2. Related Work

With the spot price of the commodity as the primary state variable, a significant amount of recent research on commodity futures has proposed to use the convenience yield as the second state variable, particularly in the context of energy commodities. Some authors view the convenience yield as an exogenous stochastic process (for example, Gibson and Schwartz 1990, Schwartz 1997) while assuming a constant price volatility. In contrast, Routledge et al. (2000) propose an equilibrium model for storable commodities in which the convenience yield becomes an inventory-dependent endogenous variable.

Despite its inobservability, the stock-dependent convenience yield has become a popular variable for the explanation of different shapes of forward curves. In contrast, little has been done in the direction of explicitly modeling the volatility of the commodity spot price as an inventory-dependent variable. The limited amount of empirical research on agricultural commodity inventory has probably stemmed from the difficulty of identifying comprehensive and accurate data, yet most empirical and theoretical research concurs in perceiving commodity price volatility as a decreasing function of stocks. Analyzing the dependence between the current price and the expectation of futures prices at a given inventory level, Deaton and Laroque (1992) find that the conditional variance of prices increases with the current price. As the current price is a decreasing function of available inventory, their model implies that price volatility decreases with higher stocks. Statistical studies performed by Fama and French (1987) on futures spreads in a number of commodity futures markets provide empirical evidence of this property, both cross-sectional and over time. Williams and Wright (1991) show that the presence of stockouts creates a nonlinear response in commodity prices and affects the variance of the commodity price.

The property of storable commodity price volatility decreasing with availability can be traced back to papers written by economists in the 1980s. Reagan (1982) considers a monopolist firm facing uncertain demand and finds that market prices react more strongly to demand disturbances when inventories are exhausted than when they are high. The result is similar in the case of a competitive industry, as shown in Reagan and Weitzman (1982).

Ghosh et al. (1987) present a discrete-time model in which price and stock movements are jointly caused by shifts affecting the supply and demand of the commodity and find that the conditional price volatility is inversely proportional to the level of stock held. Lowry et al. (1987) consider a quarterly model applied to soybeans and exhibit that the coefficient of variation of soybean price remains about the same for the
first three quarters and then increases significantly in the fourth one, with the current-year crop being harvested in the first quarter of the following year.

Although the observation of decreasing commodity price volatility with higher inventories is widely admitted, few precise tests of this property have been performed, probably due to the difficulties of getting data on inventories. Fama and French (1987) conduct a vast empirical study of the effect of storage on spot and futures prices and price volatility, with a proxy for inventory constructed out of futures contracts. Fama and French (1988) use the same representation of the inventory and conclude that metal spot prices are more variable than futures prices when inventory is low. Ng and Pirrong (1994) also analyze metal prices and find that spot and forward return variances vary directly with inventory. In all these interesting studies, no real inventory data have been used, providing only indirect evidence of the inverse relationship between inventory and volatility. One of our goals in this paper is to conduct in the context of soybean—a major agricultural commodity—the empirical analysis on observed inventory data.

3. Empirical Analysis of Soybean Prices and Inventories

The United States, Brazil, and Argentina are currently the three biggest soybean-producing countries. For decades, the United States was the world’s leading exporter of soybeans, but Argentina and Brazil have made great inroads in recent years. Since 1990, soybean production has more than doubled in Argentina and Brazil, while it has increased by about 42% during the same period in the United States. All soybean transactions, including the forward contracts with physical delivery in Europe, are tied to CBOT prices, making these members the world soybean benchmark. Let us note that physical delivery of CBOT futures taking place in a facility in Illinois does not make this location particularly special for soybean because a very small fraction of futures contracts are physically unwound at maturity (as in all commodity futures markets). To some extent, Rotterdam, which is the leading port of entry for soybean into Europe, plays a more important geographical point in terms of soybean release because the European Union is the world’s largest soybean importer (and accounted in the marketing years 1998 and 1999 for more than 35% of world soybean imports).

For the inventory analysis, our database consists of annual, quarterly, and monthly data. The annual data are provided by the United States Department of Agriculture (USDA) and include the world ending stock and those of the main producing countries over the period 1974–1999 (26 observations). The U.S. inventory represents a large part of the worldwide number, the correlation coefficient between the world stock and the U.S. stock being 0.9065.

Besides the “ready-for-use” USDA world and U.S. stock database, we downloaded national data on the U.S., the Brazilian, and the Argentine stocks, respectively, from the websites of the USDA, the Brazilian Association of Vegetable Oil Industries (ABIOVE), and the Secretaría de Agricultura, Ganadería, Pesca y Alimentos or Secretary of Agriculture, Livestock, Fisheries and Food (SAGPyA) in Argentina. All stock data are expressed in thousands of tons.

The data on the U.S. ending stocks consist of quarterly observations going from December 1989 to August 1999. The ending stock data are available in a monthly form for Brazil and Argentina, respectively, from February 1993 to 2001, and from September 1967 to December 2001. We estimate quarterly world stock data by summing national stock figures whenever all are available. Thus, we obtain this number from March 1, 1993, to September 1, 1999 (with the convention that the first quarter of year N includes December N − 1, January N, and February N), or 27 observations.

The U.S. stock data being only available in a quarterly form (March 1, June 1, etc. . . .), we constructed the figures for the first day of other months by interpolating linearly those of the two nearest months. For example, if I3 and I6, respectively, denote the inventory figures as of March 1 and June 1, then the estimate ˆI4 for April 1 is given by ˆI4 = (2I3 + I6)/3. As shown in Figure 1, the U.S. stock decreases in a remarkably linear manner (see the zoomed picture on Figure 1C) from December 1 to September 1, then increases until December 1—hence the validity of the interpolation method. Summing observed or estimated monthly data for the United States, Brazil, and Argentina provides the monthly world ending stocks from February 1993 to August 1999 (79 observations).

Table 1 reports some summary statistics about the inventory variable, including the ADF and Phillips–Perron tests for unit roots. The statistics are rarely significant, indicating that inventories are not stationary. In addition, the intercepts that are included in these tests but not reported here are significantly positive. Hence, the inventory series trends upward, in agreement with the growth observed both in world production and demand over the last decades.

2 For instance, the massive imports of soybean arriving in Europe through the harbors of Rotterdam and Saint-Nazaire now come nearly exclusively from South America. In 1999, the United States, Brazil, and Argentina accounted, respectively, for 45.17%, 21.39%, and 13.26% of the world production. As a comparison, these shares were 60.62%, 18.12%, and 0.89%, respectively, in 1974. A huge center for soybean processing, and belonging to the agri-food/commodity trading U.S. company Cargill, is based in Saint-Nazaire, a harbor on the Atlantic Ocean in France.
Figure 1  World and U.S. Ending Inventory over the 1974–1999 Period Under Various Timescales

Note. Figure 1A represents the world and U.S. annual ending inventory over the period 1974–1999. Figure 1B depicts the world and U.S. inventory during the period February 1993 to August 1999. In Figure 1C, the quarterly U.S. inventory is zoomed over the period Q4, 1994, to Q3, 1996, to show its quasi-perfect linear changes within quarters.

Table 1  Summary Statistics on U.S. and World Inventory

<table>
<thead>
<tr>
<th>Descriptive statistics</th>
<th>Annual data</th>
<th>Quarterly data</th>
<th>Monthly data</th>
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<tbody>
<tr>
<td></td>
<td>World inventory</td>
<td>U.S. inventory</td>
<td>World inventory</td>
</tr>
<tr>
<td># Observations</td>
<td>26</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td>Mean</td>
<td>10,979</td>
<td>7,252</td>
<td>33,049</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>2,965</td>
<td>2,667</td>
<td>15,011</td>
</tr>
<tr>
<td>Maximum</td>
<td>18,088</td>
<td>14,580</td>
<td>64,001</td>
</tr>
<tr>
<td>Minimum</td>
<td>5,779</td>
<td>2,801</td>
<td>9,508</td>
</tr>
<tr>
<td>Z/SLrho</td>
<td>−15.442</td>
<td>−14.7351</td>
<td>27.5728</td>
</tr>
<tr>
<td>OLS t test</td>
<td>−2.2073</td>
<td>−2.267</td>
<td>−2.6708</td>
</tr>
<tr>
<td>OLS F test</td>
<td>3.1126</td>
<td>4.0343</td>
<td>3.5733</td>
</tr>
<tr>
<td>Z1</td>
<td>−3.1577</td>
<td>−3.1057</td>
<td>−7.6861***</td>
</tr>
</tbody>
</table>

*(respectively, ** and *** ) signals a number significant at the 10% (respectively, 5% and 1%) confidence level.
Turning to soybean prices, our data consist of daily nearby futures prices (classical proxies for spot prices in commodity markets) from June 3, 1974, to October 31, 1989, and from January 2, 1990, to November 1, 2000, as well as weekly prices of all traded futures from June 7, 1974, to October 30, 1989, and from January 5, 1990, to October 26, 2000. The futures data are obtained from the Chicago Board of Trade (for unclear reasons, November and December 1989 are missing). Note that only five to seven maturities are observed at all dates, which is a concern for producers looking for long-term hedges and may be perceived as a limitation for our analysis; on some dates, 12 maturities are available. The information contained in the original CBOT data for each contract times series includes daily open, high, low, close, and settlement futures prices, as well as open interest; all prices are expressed in thousandths of a dollar. A futures contract begins to trade one and a half years prior to its maturity. We work with the settlement values of futures prices because they are classically viewed as the representative numbers for each trading day.

The data have been reorganized to be more amenable to the analysis. To build time series long enough to provide a consistent fit, we classify the futures contracts according to their time to maturity. That is, all nearby futures prices constitute a same and unique time series, as do the second-nearest futures prices and so forth. Leaving aside the discussion of complex rules on settlement and delivery periods, we consider that each futures contract matures the last day on which its price is recorded. Table 1 reports some key statistics about the U.S. and world inventories during the period February 1993 to August 1998.

To provide a general idea of the behavior of soybean prices, we depict in Figure 2 the nearby futures series over the period June 7, 1974, to October 26, 2000. We use a linear interpolation for the missing values in November and December 1989. Figure 2 shows that prices do not exhibit any obvious upward or downward trend over long time periods, an observation previously made for other commodities (see Bessembinder et al. 1995 or Pindyck 2001). A closer analysis of Figure 2 shows an annual seasonality in the pattern of the nearby futures price, with two local maxima and two local minima (this feature is also observed in Richter and Sorensen 2000). The global maximum is attained in July about two months prior to the beginning of the U.S. harvest period, while the global minimum is reached at the end of the U.S. harvesting. The local minimum is reached during April, at the end of the Brazilian harvesting. An analysis on the nearby futures volatility leads to a similar pattern. Because the impact of the U.S. harvest is much more important than that of the South American one, we will model the seasonality in soybean price and volatility by deterministic functions with an annual periodicity.

Table 2 provides the mean values of the series of the five contracts with shortest time to maturity and denoted as $F_1, F_2, \ldots, F_5$, together with the average values of these residual maturities.

Moreover, as is classically done in the financial literature, we define returns as changes in log prices and compute these returns over weekly time periods for the five series. The first four moments of these series are displayed in Table 3, with means and standard deviations expressed on an annual basis. The skewness is never significantly different from 0, but the kurtosis is much higher than 3, a standard evidence of nonnormality; in fact, the kurtosis is greater than 10 over the second period, in agreement with the sharp price fluctuations observed in the second half of the last decade. We can notice that the so-called “Samuelson effect,” stating that futures
price volatilities decrease when maturities increase, is depicted in Table 3. Finally, the close-to-zero values of the mean returns for the five futures contracts over long horizons argue in favor of the choice of a mean-reverting process for the spot price.

We now introduce the scarcity variable, denoted as $s$ and defined as inverse inventory at time $t$. If $l$ denotes the world stock at the end of period $t$, then the corresponding scarcity is $s = 1/l$. To understand the impact of inventories on volatility, we run the following regression:

$$
\sigma_t = \alpha + \delta t + \beta s_{t-1} + \epsilon_t,
$$

where $t$ denotes the period (year, quarter, or month), $s_{t-1}$ is the value of scarcity at the end of period $t - 1$, and $\sigma_t$ is the standard deviation of the nearby returns over period $t$. The presence of the deterministic drift in the right-hand side of (1) is meant to account for a possible trend in either the volatility or scarcity series. The constant $\beta$ should be positive if high inventories do reduce the volatility. It is worth noticing that the simple inverse function used to derive scarcity from inventory has the property of being both decreasing and convex. It captures the intuitive notion that when stocks are low, an additional unit of inventory will have more effect on volatility than when they are abundant.

It is clear from Figure 1 that quarterly and monthly inventories are highly seasonal, with lows in late summer. The estimated volatility of the nearby returns is also seasonal (with peaks in the third quarter just before a large amount of uncertainty is resolved by the U.S. harvest that takes place in October and November).

To make the results of the quarterly and monthly regressions clearer to interpret, we deseasonalize both $\sigma_t$ and $s_{t-1}$. We apply a moving average filter specifically designed to eliminate the seasonal component (for more details, see Brockwell and Richard 1996). Tests for unit roots show that the series of annual volatility and scarcity are stationary, as well as the deseasonalized series of quarterly and monthly volatility and scarcity. This ensures the validity of the regressions based on Equation (1).

We perform Regression 1 for different time granularities and report the results in Table 4. For instance, in Regression 1, $s_{t-1}$ is the world scarcity level at the end of year $t - 1$ in the USDA annual database (going from 1974 to 1999), and $\sigma_t$ represents the standard deviation of the daily returns in year $t$ (going from 1975 to 2000). Similarly, in Regression 3 (respectively, Regression 5) $s_{t-1}$ stands for the world scarcity level at the end of quarter (respectively, month) $t - 1$ in the data reconstructed by aggregation; $\sigma_t$ represents the standard deviation of the daily returns in quarter (respectively, month) $t$ going from Q2, 1993 to Q4, 1999 (respectively, from March 1993 to September 1999). The analysis is repeated with U.S. stock figures to determine the extent to which U.S. inventories specifically influence soybean price volatilities; the results are displayed in Regressions 2, 4, and 6.

The $F$-statistics and the standard errors for the coefficients in Table 4 indicate that relationship (1) cannot be rejected at the 1% significance level and the coefficient $\beta$ is positive at the same confidence level, regardless of the data-sampling frequency and the use of U.S. or world inventory. For instance, the $F$-statistic in Regression 5 is equal to 17.58, while the 1% critical value is only 4.05. Moreover, the coefficients obtained in the quarterly and monthly regressions are remarkably close, which proves the robustness of the relationship described in Equation (1). The slope coefficients are lower, however, in Regressions 1 and 2, possibly due to the mixture of the northern
hemisphere and southern hemisphere in USDA world stock data. The USDA world stock data are obtained as a sum of ending stock numbers of countries with different local marketing years, and hence tend to underestimate the real world stock level at any point in time. An adjustment has recently been made by the USDA, but covers only the very recent years (which explains our choice of ending our analysis in 1999 in order not to mix different types of measurement). The $R^2$ statistics indicate that the world stocks have a higher explanatory power than the U.S. stocks at the quarterly and monthly frequencies. This shows that the addition of Brazilian and Argentine inventories to U.S. inventories does improve the accurateness of the estimated inventory data. From all perspectives, we can conclude that not only is volatility inventory dependent, but also that the functional representation proposed in Equation (1) is satisfied at a highly significant level.

Continuing our empirical investigations, we display in Figures 3A–3C the soybean (deseasonalized) realized price volatility at time $t$ used in Regression 1, together with the quantities $\hat{\alpha} + \hat{\delta} t + \hat{\beta} s_{t-1}$, where $s_{t-1}$ denotes the world or U.S. scarcity level at time $t - 1$ (also used in Regression 1); $\hat{\alpha}$, $\hat{\delta}$, and $\hat{\beta}$ are the estimated regression coefficients reported in Table 4.

It appears that the time patterns of historical volatilities are closely matched by the estimated values, whether one uses the world or the U.S. stock levels. This positive correlation between volatility and scarcity is observed over long and short time periods. In Figure 3A, for instance, increases (or decreases) in scarcity almost always correspond to increases (or decreases) in volatility. Except for 1982, low levels in inventories always lead to volatility peaks, as seen in particular at the end of the years 1977, 1984, 1989, 1994, and 1997.

Figure 3B displays the quarterly scarcity comovements with volatility: The peak of volatility in Q4, 1997 is well captured by the upward moves in both world and U.S. scarcities. Similarly, the fall in volatility in Q4, 1999, is associated with a downward move in the two scarcity numbers. As mentioned earlier, the world and U.S. inventories have lows in August–September, and the realized volatility attains its highs at the same time. In Figure 3C, we observe that the deseasonalized volatility tends to be low in September, as do the deseasonalized scarcities. This proves that even when we take away the seasonality, scarcity continues to be a good proxy for volatility, which shows the robustness of the property.

Another noteworthy result is the important effect of the U.S. stocks on the soybean price volatility relative to the world stocks as shown in Figures 3A–3C. The U.S. stocks have almost the same impact on the volatility as the world stocks. Note that the U.S. scarcity does not totally capture volatility lows in Q4, 1998; apart from this time period, our representation of the price volatility in terms of the U.S. stock level seems quite satisfactory. Furthermore, Figure 3C indicates that when inventories are low, the price volatility is not only higher but also more volatile, which means that the volatility of volatility also decreases with inventories.

Table 4 and Figure 3 clearly exhibit the importance of U.S. production and inventory in explaining the

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Estimation of Equation (1)</th>
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<tbody>
<tr>
<td></td>
<td>Regression 1</td>
</tr>
<tr>
<td>$\sigma_t$</td>
<td>Annual nearby volatility</td>
</tr>
<tr>
<td>$s_t$</td>
<td>Annual world scarcity</td>
</tr>
<tr>
<td>Number of observations</td>
<td>26</td>
</tr>
<tr>
<td>$\hat{\alpha}$</td>
<td>0.1435**</td>
</tr>
<tr>
<td></td>
<td>(0.0514)</td>
</tr>
<tr>
<td>$\hat{\delta}$</td>
<td>−0.001</td>
</tr>
<tr>
<td></td>
<td>(0.0015)</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>1022**</td>
</tr>
<tr>
<td></td>
<td>(397)</td>
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<tr>
<td>Coefficient of determination</td>
<td>0.3228</td>
</tr>
<tr>
<td>$F$-statistic</td>
<td>5.4809**</td>
</tr>
</tbody>
</table>

**Note.** This table reports the values of the nearby futures volatility $\sigma_t$ and world scarcity $s_t$ under various timescales. $\hat{\alpha}$ denotes the estimated vertical intercept, $\hat{\delta}$ the estimated coefficient of $t$, and $\hat{\beta}$ the estimated coefficient of $s_{t-1}$ in the regression of $\sigma_t$ on $t$ and $s_{t-1}$.

**(respectively, **, ***)** indicates a 10% (respectively, 5%, 1%) confidence level.
Figure 3 Soybean Historical Volatility and Its Six Estimated Values

Note. The time series labeled “Historical volatility” represents the standard deviation of soybean nearby futures daily returns over one-year (respectively, one-quarter, one-month) periods in Figure 3A (respectively, 3B, 3C). The time series labeled “Estimated volatility $i$” $(i = 1, 2, \ldots, 6)$ represents the values estimated from “Regression $i$” defined in Table 4. For instance, Figure 3A represents the annual historical volatility and its estimated values obtained from regressions on the inverse of the annual world stock (Regression 1) and on the inverse of the annual U.S. stock (Regression 2), with inclusion of a linear time trend. The analogous quantities for quarterly and monthly frequencies are displayed in Figures 3B and 3C.
volatility of soybean prices in international markets. These properties being established, we now turn to a state-variable approach to the dynamics of soybean forward curves.

4. The Two-State-Variable Model

We first build a two-state-variable model that does not invoke scarcity. We denote the commodity spot price by $S_t$ with $t \in [0, \infty)$. The dynamics of the spot price are classically defined through $\ln S_t$ as follows.

**Assumption 1.** The logarithm of the spot price is a sum:

$$\ln S_t = h_t + X_t, \quad (2)$$
$$h_t = \eta \sin(2\pi(t + \varphi)), \quad (3)$$

where the deterministic term $h_t$ represents seasonality in agricultural commodity prices and $X_t$ is the deseasonalized stochastic component.

**Assumption 2.** The dynamics of the stochastic component of the spot price are driven under the real probability measure by the stochastic differential equation

$$dX_t = (\kappa(v_{1t} - X_t) + \lambda X_t \sigma e^{\psi_t}) dt + \sigma e^{\psi_t} dW^X_t, \quad (4)$$

where

$$\psi_t = \theta \sin(2\pi(t + \zeta)) \quad (5)$$

is a deterministic term allowing us to represent seasonal pattern in volatility; $v_{1t}$ is the short-term mean of $X_t$. $\sigma$ is a positive constant and $\lambda X_t$, the market price of commodity risk, is assumed to be constant.

The constants $\varphi$ and $\zeta$ in Equations (3) and (5) allow us to adjust the maximum of $h_t$ and $\psi_t$ to the choice of the time origin. To insure the uniqueness of the parameters, we impose the constraints $\varphi \in [-0.5, 0.5)$, $\zeta \in [-0.5, 0.5)$, $\eta > 0$, $\theta > 0$ with the maturity of the January futures contract as the time origin.

**Assumption 3.** The evolution of the short-term mean is described by the stochastic differential equation

$$dv_{1t} = (a_1(b_1 - v_{1t}) + \lambda_1 \xi_1 e^{\psi_t}) dt + \xi_1 e^{\psi_t} dW^{v_{1t}}, \quad (6)$$

where the constant $b_1$ represents the long-term value of the short-term mean, and $\xi_1$ its seasonal adjusted volatility. $\lambda_1$ is the risk premium on short-term mean uncertainty.

There is a correlation $\rho_{12}$ between the Brownian motions $W^X_t$ and $W^{v_{1t}}$. All parameters are supposed to be time independent; $\kappa$, $\sigma$, $a_1$, and $\xi_1$ are positive.

As expressed in Equations (4) and (6), we assume that both state variables follow mean-reverting processes. The mean reversion reflects the self-adjustment behavior in supply-demand patterns that influence the stochastic component of the spot price as well as its short-term mean. The Ornstein-Uhlenbeck process has the merit of providing this feature while being very tractable from a mathematical standpoint; negative values are excluded because we are modeling log prices.

We assume no arbitrage opportunities; markets are complete because we have more instruments than sources of risk—hence, the existence and unicity of a risk-neutral probability measure $Q$. The form proposed in Assumptions 2 and 3 for the dynamics of $X_t$ and $v_{1t}$ implies that under this measure $Q$, the dynamics of the stochastic component of the spot price and its short-term mean are driven by the following stochastic differential equations:

$$dX_t = \kappa(v_{1t} - X_t) dt + \sigma e^{\psi_t} d\hat{W}^X_t, \quad (7)$$
$$dv_{1t} = a_1(b_1 - v_{1t}) dt + \xi_1 e^{\psi_t} d\hat{W}^{v_{1t}}, \quad (8)$$

where $\hat{W}^X_t$ and $\hat{W}^{v_{1t}}$ are Brownian motions under the measure $Q$.

Because the futures price is a Q-martingale, Assumptions 1, 2, and 3 yield the price $F^T_t$ at time $t$ of the futures contract maturing at $T$:

$$F^T_t = E_Q(S_T / \mathcal{F}_t) = e^{h_t + A(t, T) + B(t, T)(\ln S_t - \hat{a})_u + C(t, T)v_{1u}}, \quad (9)$$

where $\mathcal{F}_t$ represents the information available at time $t$, and the quantities involved in the futures price $F^T_t$ have the following expressions:

$$A(t, T) = a_1 b_1 \frac{\kappa}{\kappa - \hat{a}_1} \left(1 - \frac{e^{-\hat{a}_1(T-t)}}{\hat{a}_1} - \frac{1 - e^{-\kappa(T-t)}}{\kappa}\right) - \frac{1}{2} \int_t^T e^{\psi_u} \left(B(u, T)^2 \sigma^2 + 2B(u, T)C(u, T)\right) \rho_{12}\xi_1 e^{\psi_u} + C(u, T)^2 \xi_1^2 \right) du, \quad (10)$$

$$B(t, T) = e^{-\kappa(T-t)}, \quad (11)$$

$$C(t, T) = \frac{\kappa}{\kappa - \hat{a}_1} \left(e^{-\hat{a}_1(T-t)} - e^{-\kappa(T-t)}\right).$$

These computations are fairly standard and described in detail in Appendix 1 (available online at http://mancsi.pubs.informs.org.e enhancements.html). The integral in Equation (10) is not available in closed form; we compute it numerically using the trapezoidal approximation.

Let us note two properties obtained in our model: The coefficient $B(t, T)$ multiplying $\ln S_t$ in the expression of $F^T_t$ described by Equation (9) is a positive, decreasing, and convex function of time to maturity. Hence, for a constant short-term mean $v_{1u}$, we see that a positive shock on the spot price will not only shift up the whole forward curve, but also result in a greater backwardation (i.e., short-term futures prices higher than long-term ones). The coefficient $C(t, T)$ is
also positive. It increases for
\[ T - t \in \left( 0, \frac{\ln(\kappa/a_1)}{\kappa - a_1} \right) \]
then decreases, hence reducing (as expected) the impact on \( F_t^T \) of the short-term mean term \( v_{1,t} \).

5. The Three-State-Variable Model
We now augment the model with scarcity as a third state variable; as a consequence of \( \S 3 \), the variance of spot returns denoted as \( v_{2,t} \) becomes stochastic. More precisely, we translate the results of \( \S 3 \) into:

**Assumption 4.** The variance of the spot return is given by the following relationship:
\[ v_{2,t} = (\alpha + \beta s_t)^2, \quad (12) \]
where \( \alpha \) and \( \beta \) are constant numbers, with \( \beta \) positive.

**Assumption 5.** The dynamics of the stochastic component of the spot price, its short-term mean and its variance are driven under the real probability measure \( P \) by the following stochastic differential equations:
\[
\begin{align*}
\frac{dX_t}{X_t} &= (\kappa(v_{1,t} - X_t) + \lambda_2 e^v_{2,t}) dt + e^{\psi} \sqrt{v_{2,t}} dW_t^X, \\
\frac{dv_{1,t}}{v_{1,t}} &= (a_1(b_1 - v_{1,t}) + \lambda_1 \sigma_1 e^v_{1,t}) dt + \sigma_1 e^v_{1,t} \sqrt{v_{2,t}} dW_t^{v_1}, \\
\frac{dv_{2,t}}{v_{2,t}} &= (a_2(b_2 - v_{2,t}) + \lambda_2 \sigma_2 v_{2,t}) dt + \sigma_2 \sqrt{v_{2,t}} dW_t^{v_2},
\end{align*}
\]
where the parameters \( a_2, b_2, \sigma_2 \) and \( \lambda_2 \) are positive and \( \lambda_2 \) is the market price of volatility risk (see, for instance, Heston 1993). \( \rho_{12} \) (respectively \( \rho_{13}, \rho_{23} \)) denotes the correlation coefficient between the Brownian motions \( W_t^X \) and \( W_t^{v_1} \) (respectively, \( W_t^X \) and \( W_t^{v_1} \); \( W_t^{v_1} \) and \( W_t^{v_2} \)). All parameters are supposed to be time independent.

Because we have at all times five to seven liquid futures contracts compared to three sources of randomness, the situation of completeness still prevails, and we can state that under the unique pricing measure \( Q \), the dynamics of the stochastic component of the spot price, its short-term mean and its variance are driven by the following stochastic differential equations:
\[
\begin{align*}
\frac{dX_t}{X_t} &= \kappa(v_{1,t} - X_t) dt + e^{\psi} \sqrt{v_{2,t}} d\hat{W}_t^X, \\
\frac{dv_{1,t}}{v_{1,t}} &= a_1(b_1 - v_{1,t}) dt + e^{\psi} \sigma_1 \sqrt{v_{2,t}} d\hat{W}_t^{v_1}, \\
\frac{dv_{2,t}}{v_{2,t}} &= a_2(b_2 - v_{2,t}) dt + \sigma_2 \sqrt{v_{2,t}} d\hat{W}_t^{v_2},
\end{align*}
\]
where \( b_2 \) and \( \sigma_2 \) are, respectively, the long-term mean and the volatility of the variance of the log price.

Let us make some comments on the model described by Equations (16) to (18).

(a) The choice of a square-root process for the process for the volatility in Equation (20) ensures positivity of the solution, a necessary feature for volatility, while mean reversion implies bounded values, as it is the case for the volatility of any asset.

(b) The integration of Equations (16) and (18) (Ornstein-Uhlenbeck process \( S_t \) with a stochastic variance \( v_{2,t} \) driven by a square-root process) is not simple. In fact, the distribution of \( S_t \) is fairly complex because it is related to squared-Bessel processes. (For more details, see Geman and Yor 1993.) However, we just need to compute its expectation at date \( T \) to obtain the futures prices \( F_t^T \) exhibited below. This is more straightforward because it only involves the expectation of the integral between \( t \) and \( T \) of the variance and Fubini’s theorem provides the answer fairly easily.

(c) The fact that \( v_{2,t} \) is an observable quantity implies that the Kalman filter procedure will only involve normally distributed quantities, a situation where it has been proven quite robust.

(d) The choice of the random term in the dynamics of \( v_{1,t} \) described in Equation (17):
- Reflects the impact on the short-term mean of a shock in the spot price volatility;
- Allows adjustment of the total variance of \( v_{1,t} \) through the coefficient \( \sigma_1 \);
- Incorporates in the Brownian motion \( W_t^{v_1} \) the “direction” of the randomness of \( v_{1,t} \) in its own right, with correlation coefficients \( \rho_{12} \) and \( \rho_{23} \), with \( W_t^X \) and \( W_t^{v_2} \) being free parameters;
- Ensures that the variance-covariance matrix in the Kalman filter estimation is linear in all variables.

Our representation of the spot price has the merit of exhibiting both a mean-reverting behavior and stochastic volatility. Moreover, we have demonstrated that this volatility is simply related to inventory, a quantity which, in contrast to volatility, has the merit of being observable.

Assumptions 1, 2, and 5 imply the following dynamics for the spot price under the measure \( Q \):
\[
\begin{align*}
\frac{dS_t}{S_t} &= \kappa \left[ \left( v_{1,t} + 1 \right) \frac{e^{\psi} \sqrt{v_{2,t}} d\hat{W}_t^X}{2} + \frac{dh_t}{dt} \right] + h_t - \ln S_t \right] dS_t dt \\
&+ e^{\psi} \sqrt{v_{2,t}} S_t d\hat{W}_t^X,
\end{align*}
\]
Hence, in turn, the futures price value is expressed as
\[
F_t^T = E_Q(S_T/\hat{F}_t) = e^{\beta_T + G(t, T) + B(t, T)X_t + C(t, T)v_{1,t} + D(t, T)v_{2,t}},
\]
or
\[
F_t^T = E_Q(S_T/\hat{F}_t)
= e^{\beta_T + G(t, T) + B(t, T)(\ln S_t - h_t) + C(t, T)v_{1,t} + D(t, T)v_{2,t}},
\]
and

\[ G(t, T) = a_1 b_1 \frac{\kappa}{\kappa - a_1} \left( \frac{1 - e^{-a_1(T-t)}}{a_1} - \frac{1 - e^{-\kappa(T-t)}}{\kappa} \right) + a_2 b_2 \int_t^T D(u, T) \, du, \]

where \( D(t, T) \) is the solution of the ordinary differential equation

\[ D(t, T) = -\frac{1}{2} \sigma_D^2 D(t, T) \frac{D(t, T)^2}{2} + D(t, T) \left( a_2 - \rho_{13} \sigma_2 e^{\theta_0} B(t, T) \right) - \rho_{23} \sigma_1 \sigma_2 C(t, T) e^{\theta_0} \]

\[ - \frac{1}{2} B(t, T)^2 e^{2\theta_1} - \rho_{12} \sigma_1 B(t, T) C(t, T) e^{2\theta_1} \]

\[ - \frac{1}{2} C(t, T)^2 \sigma_1^2 e^{2\theta_1} \]

with terminal condition \( D(T, T) = 0 \).

The solution to the ordinary differential Equation (23) is not available in closed form. It is, however, easily obtained numerically with high precision and speed using, for example, Runge–Kutta methods. Again, we compute the integral in (22) numerically using the trapezoidal rule.

The expressions for \( B(t, T) \) and \( C(t, T) \) are the same as before.

Note from Equation (20) that stochastic volatility may affect the shape of the forward curve directly and indirectly—directly by the presence of \( v_{2i} \), indirectly as follows: If there exists a positive correlation between \( X_i \) and \( v_{2i} \), a positive shock on volatility will, on average, go together with a positive shock on the stochastic component of the spot price. The impact on \( F^T \) of this shock decreases for distant maturities; hence, the backwarcad effect becomes more accentuated.

The volatility of returns on a futures contract \( F^T \) can be derived from (20):

\[ \sigma_f^2(t, T) = v_{2i} (B(t, T)^2 e^{2\theta_i} + C(t, T)^2 \sigma_1^2 e^{2\theta_1} + D(t, T) \sigma_D^2 + 2B(t, T) C(t, T) \rho_{12} \sigma_2 e^{\theta_1} + 2B(t, T) \rho_{23} \sigma_1 \sigma_2 e^{\theta_1} + 2C(t, T) D(t, T) \rho_{23} \sigma_1 \sigma_2 e^{\theta_1}). \]  

Taking \( T = t \), we recognize the spot price volatility

\[ \sigma_f^2(t, t) = v_{2i}. \]  

6. Statistical Data and Estimation

In this section, we estimate the models developed in §§4 and 5 to soybean futures and inventory data. One of our goals is to determine whether the scarcity variable has a significant explanatory power.

6.1. The Kalman Filter

We apply the Kalman filtering procedure to the time series of five maturity futures prices (\( N = 5 \) in the following) described above and observed weekly from March 5, 1993 to August 19, 1999. These five maturities also represent the most liquid ones and constitute the sample of futures contracts over which both models are estimated.

The Kalman filter is a recursive method (see the details in Appendix 2, available online) for computing estimates of unobserved state variables, usually normally distributed, through observations of quantities that depend on these state variables. The evolution of the state variables is described by the “transition equation” while the relationship between the state variables and the observables (futures prices, in our setting) is called the “measurement equation” (see Harvey 1989). The optimization procedure consists of maximizing a total log-likelihood function that is the sum of the individual conditional log-likelihood terms (note that in the case of normality, conditional distributions are also normal). The required inputs are the initial values at \( t = 0 \) of the unobserved state variables denoted \( Z_0 \) and their variance-covariance matrix \( P_0 \). Harvey (1989) suggests choosing \( P_0 = 0 \) and optimize the choice of \( Z_0 \) through the so-called “Rosenberg algorithm;” this is the path we adopted.

There are two reasons that motivate our choice of the Kalman filter. First, distant contract prices (beyond the five closest ones) are not available at every observation date. The further away the maturity of a contract, the fewer available observations (in sharp contrast, for instance, with the situation of interest rates for which numerous instruments are observed across a large spectrum of maturities). Second, there is no clearly defined underlying spot price pertaining to our soybean futures data as in most agricultural commodity markets. With two unobservable state variables and modest cross-sectional data, the Kalman filter approach is viewed today as quite appropriate.

In the two-factor model, both state variables are unobservable and are estimated through the Kalman filter. In the three-factor model, only the stochastic component of the spot price and its short-term mean are unobservable; the scarcity variable is directly obtained as the inverse of the inventory numbers. Recall that the variance of the spot return is given by Equation (12):

\[ v_{2i} = (\alpha + \beta s_i)^2, \]

where the parameters \( \alpha \) and \( \beta \) are estimated by maximizing the likelihood function attached to the Kalman filter, together with the other parameters of the three-state-variable model. Regarding the resolution at the monthly frequency of the three-factor model, we will
follow two alternative routes as explained below. Note that, consistently with the previous sections, we impose the constraint $\beta > 0$ in the maximization of the likelihood.

6.2. Empirical Results

Table 5 gives the estimated parameters and standard errors of the two- and three-state-variable models and shows that the estimated values of the common parameters between the two models are comparable. We want to mention that to overcome the problem of missing data in the monthly inventory observations, we approached the three-state-variable model in two manners:

(a) by creating estimates of these data through linear interpolation of quarterly ones, and

(b) by treating the monthly scarcity as another state variable for which the measurement equation has the simple form

$$s(t) = \tilde{s}(t) + \epsilon_t,$$

where $\tilde{s}(t)$ is the (observable) estimator of $s(t)$ obtained by the interpolation method described in (a). The numbers obtained for the parameters under the two procedures are totally undistinguishable, which is not surprising given the linearity exhibited by the inventory behavior within a quarter (see Figure 1C).

The parameters $\sigma_2$ and $\beta$ are of particular interest because they justify the introduction of the scarcity variable (and in turn a stochastic volatility) in the three-factor model. Both parameters are found to be significant even at the 1% level. Because $\alpha$ and $\beta$ are significantly positive, Equation (12) implies that a rise in scarcity results in an increase in volatility.

We wish to emphasize that the estimated value of the coefficient $\beta$ (7.927) is quite close to the value of 6.963 found in Regression 5, Table 4, while these two numbers were obtained through totally different procedures (monthly regression of price volatility on scarcity versus Kalman filter estimation of the three-state-variable model). The speeds of adjustment coefficients $\kappa$, $a_1$, and $a_2$ are highly significant, justifying the use of the mean-reverting processes to model the state variables. The coefficient $\kappa$ is much smaller than $a_1$, which indicates that the short-term mean $\nu_{10}$ corrects any deviation much faster than the stochastic component of the spot price. For example, in the two-state-variable model, the estimate for $a_1$ is equal to 1.6866, which implies that one-half of any deviation from the average short-term mean is expected to be corrected in 0.41 years. Most of the other parameters are also highly significant, except for the correlations $\rho_{12}$ and $\rho_{23}$ and for the risk premia $\lambda_X$ and $\lambda_1$.

The authors thank the referee for this suggestion. It allowed them to avoid the mathematical complexity attached to the transition equation satisfied by the scarcity process ($st$), while fully reflecting the observed and nonobserved quantities under analysis.

### Table 5

#### Estimated Parameters of the Two- and Three-State-Variable Models

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Two-factor model</th>
<th>Three-factor model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>0.7529 (0.0112)</td>
<td>0.6831 (0.0049)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.241 (0.0001)</td>
<td>0.193 (0.0001)</td>
</tr>
<tr>
<td>$a_1$</td>
<td>1.6866 (0.0440)</td>
<td>1.8327 (0.0224)</td>
</tr>
<tr>
<td>$b_1$</td>
<td>8.8526 (0.0002)</td>
<td>8.76 (0.0023)</td>
</tr>
<tr>
<td>$\xi_1$</td>
<td>0.5781 (0.0109)</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.0001 (0.0000)</td>
<td>0.0000 (0.0000)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>7927 (109)</td>
<td></td>
</tr>
<tr>
<td>$\lambda_X$</td>
<td>-0.0006 (0.2846)</td>
<td>-0.0005 (4.1892)</td>
</tr>
<tr>
<td>$\lambda_1$</td>
<td>-0.4853 (0.3759)</td>
<td>-2.6368 (4.4689)</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>0.0014 (0.0001)</td>
<td>0.0013 (0.0001)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>0 (0.0000)</td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0 (0.0000)</td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>$\sigma_1$</td>
<td>0.0001 (0.0000)</td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>$\sigma_2$</td>
<td>0 (0.0000)</td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>$\sigma_3$</td>
<td>0 (0.0000)</td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>$\rho_{12}$</td>
<td>0.0012 (0.0101)</td>
<td>0.0008 (0.0100)</td>
</tr>
<tr>
<td>$\rho_{13}$</td>
<td>0.9485 (0.2497)</td>
<td></td>
</tr>
<tr>
<td>$\rho_{23}$</td>
<td>1.005 (0.1582)</td>
<td></td>
</tr>
<tr>
<td>$\eta_1$</td>
<td>0.0183 (0.0000)</td>
<td>0.019 (0.0000)</td>
</tr>
<tr>
<td>$\eta_2$</td>
<td>-0.1417 (0.0000)</td>
<td>-0.0025 (0.0003)</td>
</tr>
<tr>
<td>$\eta_3$</td>
<td>0.4612 (0.0012)</td>
<td>0.6575 (0.0021)</td>
</tr>
<tr>
<td>$\xi$</td>
<td>-0.0069 (0.0001)</td>
<td>-0.0815 (0.0001)</td>
</tr>
<tr>
<td>$\omega_1$</td>
<td>0.0015 (0.0008)</td>
<td></td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>0 (0.0000)</td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>$\omega_3$</td>
<td>0 (0.0000)</td>
<td>0 (0.0000)</td>
</tr>
<tr>
<td>$\omega_4$</td>
<td>0.0002 (0.0000)</td>
<td>0.0001 (0.0000)</td>
</tr>
<tr>
<td>$\omega_5$</td>
<td>4.391</td>
<td>4.600</td>
</tr>
</tbody>
</table>

Note: $\omega_1, \omega_2, \ldots, \omega_5$ represent the variance of measurement errors associated with the contracts $F_1, \ldots, F_5$ in the measurement equation (see Appendix 2). Standard errors are in parentheses.

A discussion of this group of parameters is in order at this point: First, $\lambda_2$ is not estimated at all because it does not appear in the transition equation nor in the measurement equation, while the risk premia $\lambda_X$ and $\lambda_1$ are nonsignificant. Schwartz (1997) also found that the risk premia related to oil, copper, and gold in these models are mostly nonsignificant. What is clear in our setting is that the relationship linking the state variables (spot price, its short-term mean, and volatility) to the observables (futures prices) prevails under the pricing measure $Q$ (see Equation (9)). Hence, we should not be surprised not to obtain risk premia $\lambda_X$, $\lambda_1$, $\lambda_2$ pertaining to the dynamics of the state variables under the real probability measure $P$. This is not a subject of concern here, as our goal is not to make predictions or forecasts of spot prices in the future.

Finally, the correlation coefficients $\rho_{12}$ (between spot price and short-term mean) and $\rho_{23}$ (between short-term mean and volatility) are nonsignificant. In contrast, the price-volatility correlation $\rho_{13}$ is estimated to be significantly positive (0.9485), in conformity with the theory of storage. The variances of measurement errors are relatively small, which is usual in this type of analysis.

To compare the two models, we can use the likelihood ratio test because the two-state-variable model is
nested in the three-state-variable model and obtained for the restrictions \( a_3 = \sigma_3 = \beta = 0 \), which form the null hypothesis \( H_0 \). Let \( \Theta_2 \) and \( \Theta_3 \) denote the maximum-likelihood estimator, respectively, for the two- and three-state-variable models, and \( L(\Theta_2) \) and \( L(\Theta_3) \) the corresponding log-likelihood values. The likelihood ratio statistic,

\[
LR = -2(L(\Theta_2) - L(\Theta_3)),
\]

is asymptotically distributed, under \( H_0 \), as a chi-square variable with a number of degrees of freedom equal to the number of restrictions. Because the weekly innovations in the state variables are approximately normally distributed, we regard the chi-square distribution as a valid approximation for a finite sample. The log-likelihoods \( L(\Theta_2) \) and \( L(\Theta_3) \) obtained from the Kalman filter on the two- and three-state-variable models are, respectively, equal to 4,391 and 4,600; thus, \( LR \) is equal to 418. Because there are three restrictions, we compare this to the critical value 7.81 at the 5% level for a \( \chi^2 \) distribution. The empirical \( LR \) being much greater than its critical value, the likelihood ratio test confirms the validity of the introduction of the scarcity variable.

As our two models contain different numbers of parameters, it would seem desirable to take this fact into account. Therefore, an alternative approach to our model selection may be based on a goodness-of-fit criterion with appropriate attention given to parsimony, such as the Akaike information criterion (AIC) or the Bayes information criterion (BIC). Let \( n_i \) \((i = 2, 3)\) denote the numbers of parameters in \( \Theta_2 \) and \( \Theta_3 \). Then,

\[
AIC_i = -2 \ln(L(\Theta_i)) + 2n_i,
\]

while

\[
BIC_i = -2 \ln(L(\Theta_i)) + n_i \ln T,
\]

where \( T \) is the number of observations. The decision rule in both cases is to select the model that minimizes the criterion statistic. Incorporating the sample size \( T = 326 \), the numbers of parameters \( n_2 = 17 \) and \( n_3 = 23 \) in Equations (27) and (28) yields

\[
AIC_2 = -8,748, \quad BIC_2 = -8,684,
\]

\[
AIC_3 = -9,154, \quad BIC_3 = -9,067.
\]

Thus, according to either criterion the three-state-variable model is superior, which supports the result of the likelihood ratio test. It is known that the BIC criterion generally favors the models with a lesser number of parameters. Accordingly, our AIC statistic results in a larger difference between the two models than the BIC one.

To perform an out-of-sample comparison between the two models, we apply the parameters obtained in sample to all futures prices available for the period of analysis (the variances of measurement errors for the out-of-sample contracts being estimated through a maximization of likelihood). Table 6 reports the prediction errors defined as the differences between observed prices and the one-week-ahead expected prices, respectively, for in-sample contracts (F1 to F5) and out-of-sample contracts (F6 to F12). The second column reports the root mean-squared errors computed individually and on the different subsets: The addition of scarcity as a state variable improves the fit of all out-of-sample contracts. Indeed, the root mean-squared error of these contracts goes from 0.0246 in the second column to 0.0224 in the fourth column, while the average absolute prediction error is reduced by more than 10% (going from 0.2154% to 0.1931% of the predicted prices). Interestingly, the last

### Table 6 Prediction Errors in the Two Models Using the World Inventory Data over the March 5, 1993 to August 19, 1999 Period

<table>
<thead>
<tr>
<th>Futures contracts</th>
<th>Two-state-variable model</th>
<th>Three-state-variable model</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Root mean-squared error</td>
<td>Average absolute prediction error in %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>In-sample contracts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F1</td>
<td>0.0437</td>
<td>0.3343</td>
</tr>
<tr>
<td>F2</td>
<td>0.0285</td>
<td>0.2412</td>
</tr>
<tr>
<td>F3</td>
<td>0.0292</td>
<td>0.2456</td>
</tr>
<tr>
<td>F4</td>
<td>0.0273</td>
<td>0.2291</td>
</tr>
<tr>
<td>F5</td>
<td>0.0292</td>
<td>0.2521</td>
</tr>
<tr>
<td>Out-of-sample contracts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F6</td>
<td>0.0244</td>
<td>0.2002</td>
</tr>
<tr>
<td>F7</td>
<td>0.0240</td>
<td>0.2020</td>
</tr>
<tr>
<td>F8</td>
<td>0.0233</td>
<td>0.2049</td>
</tr>
<tr>
<td>F9</td>
<td>0.0229</td>
<td>0.2070</td>
</tr>
<tr>
<td>F10</td>
<td>0.0285</td>
<td>0.2823</td>
</tr>
<tr>
<td>F11</td>
<td>0.0383</td>
<td>0.4227</td>
</tr>
<tr>
<td>F12</td>
<td>0.0384</td>
<td>0.4332</td>
</tr>
<tr>
<td>In-sample contracts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(F1–F5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Out-of-sample contracts</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(F7–F12)</td>
<td>0.0246</td>
<td>0.2154</td>
</tr>
</tbody>
</table>

Note: F1, . . . , F12 denote the 12 contracts in the order of expiry dates. In the three-factor model, we use the world scarcity (described in Regression 5, Table 4) and interpolate data whenever observations are missing. Both models are calibrated to the first five contracts. The parameters are then applied to the out-of-sample contracts. Both in-sample and out-of-sample prediction errors are reported. The second column displays the root mean-squared errors. The third column labeled “Average Absolute Prediction Error in Percentage” is obtained by dividing the absolute error by the futures log-price and averaging over all observations.
contract root mean-squared error is more than halved when moving from the second column to the fourth. We note that the introduction of the scarcity variable derived from the world inventory does result in a reduction of the prediction errors for the nearby futures, but to a lesser extent, an unsurprising fact because the nearby contract price is mostly driven by the spot price variable. The patterns of reduction in the errors when using U.S. inventories instead of the world ones look remarkably similar. It is worth noticing that the prediction errors in both models are very small compared to the futures prices themselves: Most of the values reported in the third and fifth columns are smaller than 0.3%.

In Figure 4 we plot the scarcity coefficient $D(t, T)$ as a function of time-to-maturity $T - t$ and the corresponding theoretical futures log-prices at some sample dates. The selected dates of observations respond to the following criteria: First, they provide the different shapes of observed forward curves (including Z-shapes, hump shapes, and tortuous curves in contango or backwardation) and the corresponding function $D$. Second, they are days of high volume relative to the remaining sample dates (over the time period March 5, 1993, to August 19, 1999). We use the parameter values implied from the in-sample fit of the three-state-variable models to compute the values of $D(t, T)$ and theoretical futures log-prices for maturities up to three years. To study the impact of scarcity on the shape of the forward curve, we indicate the scarcity level on the top of each figure in Figure 4. To facilitate the interpretation, we standardize the scarcity variable by setting its minimal value to 0 and its maximal value to 1.

If there was no seasonality in volatility ($\psi = 0$), then from Equation (25), the solution $D(t, T)$ would only depend on time-to-maturity ($T - t$), and, in turn, the shape of $D(t, T)$. In Figure 4, however, the shape of $D$ changes according to the observation time $t$ and confirms the presence of seasonality.

As shown in this figure, the function $D(t, T)$ is positive over its domain. Thus, an increase in volatility (caused by tight supply/demand conditions) results in an upward shift of the forward curve. Moreover, when time-to-maturity goes to infinity, $D$ goes to zero. This is consistent with the fact that over a long time horizon, inventory/supply adjustments can be made in response to demand shocks, resulting in a mild impact of the current scarcity on long-term contracts. The function $D$ is hump-shaped, meaning that

\[ D(t, T) \text{ is a Function of } (T - t) \text{ and the Corresponding Forward Curve at Selected Dates} \]

![Figure 4](image-url)
the effect of a rise in volatility on the prices of the contracts with maturity close to the hump is higher than that on those of shorter or longer-term contracts. There are two humps, the first one being more distinct than the second (the second hump results, in fact, from the introduction of the seasonal term $\psi_3$).

From Table 5, all correlations $\rho_{12}$, $\rho_{13}$, and $\rho_{23}$ are positive and, respectively, equal to 0.0008, 0.9485, and 0.1005. This means that a positive shock to the scarcity (or the volatility) will typically be accompanied by a positive shock to the stochastic component and its short-term mean (the indirect effect). As the coefficients $B$, $C$, and $D$ in Equation (22) are all positive, this leads to an upward shift of the whole forward curve. The behavior of $B$, $C$, and $D$ for distant maturities shows that an increase in scarcity generally results in a more backdated forward curve.

Figure 4 also shows the quality of fit of the three-state-variable model to the observed market prices. The familiar shapes of backwardation and contango do not hold for soybean forward curves because these are generally sinuous with sharp changes in the slope. The average maturities of the contracts used for estimation are below 0.8 years (see Table 2), yet our three-factor model matches remarkably out-of-sample prices well, including those whose maturity goes beyond two years. Besides the obvious capability of three state variables to better capture the level, steepness, and curvature of forward curves, another explanation is the importance of inventory in its own right to explain soybean futures prices, as has been exhibited in other storable commodity markets.

Figure 5 below displays the term structure of volatilities of futures returns and the term structure implied by the three-state-variable model for June 1993 and June 1996, two months during which they are hump-shaped. The observed volatilities are the monthly volatilities computed over the considered months. To build the theoretical term structure of volatilities, we use the parameters estimated for the three-state-variable model and the scarcity level at the beginning of the month. Note that from Equation (26), the level of the theoretical structure of volatilities depends on the level of $v_2\alpha$, but its shape does not (however, the shape of the function $\sigma^2_2(t, T)$ depends on the date $t$ because of seasonality in price and inventory). Hence, the theoretical term structures of volatilities have similar shapes in Figures 5A and 5B, only their levels differ. As in Figure 4, we indicate the scarcity number on the top of each figure, which shows that the inventory level is moderate in June 1993 and June 1996. The observed term structures of futures return volatilities in Figure 5 are hump-shaped. Tracing the observed and theoretical term structures at different instants of the year shows that the hump shapes often occur in the first half of the year (before June), a period during which the inventory level is rather high. This seems to be in agreement with the explanation given by Routledge et al. (2000) that enough inventory precludes stock-outs, inducing a positive sloping segment in the term structure of volatilities (i.e., a violation of the Samuelson effect).

7. Conclusion
We have argued in this paper that inventory plays a central role in explaining price volatility and forward curves in the case of soybeans. Using an original world inventory database, we have first shown that volatility is a linear function of inverse inventory or scarcity. This property is documented on monthly, quarterly, and annual inventories. Second, the addition to the stochastic component of the spot price and its short-term mean of scarcity as a third state variable considerably improves the quality of the estimation of a database of forward prices. The greater capacity of the three-state-variable model to fit various shapes of forward curves is particularly pronounced on distant maturities, which makes our representation quite useful in the valuation of long-term origination
contracts between producing countries and agrifood companies.

An electronic companion to this paper is available at http://mansci.pubs.informs.org/e companion.html.

Acknowledgments
This paper benefits greatly from the comments by the referee, the associate editor, and the department editor.

References


