# Curriculum of Scientific and Teaching activity of Alessandra Bernardi

## 1 Essential Data

- **Born** June 27, 1977 in Porretta Terme, Bologna, Italy;
- **Nazionality** Italian;
- **Actual Position, November 2010 – November 2012**: Marie Curie Fellow (Post Doc) at INRIA (Institut National de Recherche en Informatique et en Automatique) Sophia Antipolis - Méditerranée (Francia);
- **Belonging to**: INRIA (Institut National de Recherche en Informatique et en Automatique), Sophia Antipolis - Méditerranée (Francia);
  - **Office Mailing Address**:

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- **Guest at:**
  - Math Department on Università degli Studi di Bologna, Italy.
  - Math Department on Università degli Studi di Trento.

## 2 Education

- June 1996. Qualified at the high school Liceo Scientifico of Polo Scolastico Maria Montessori in Porretta Terme, Bologna, Italy, with 60/60.
- March,16, 2001 Graduated in mathematic at Alma Mater Studiorum of Bologna with 110/110 with praise. Thesis title: *"Schemi 0-dimensionali e forme canoniche di polinomi omogenei"*, (i.e. *"0-dimensional schemes and canonical forms of homogeneous polinomials"*). Advisor: professor Alessandro Gimigliano (University of Bologna).

  > *In this thesis there are firstly presented the relations between already solved problems like "dimensions of secant varieties of Veronese varieties", "postulations of fat points" and "Big Waring Problem". Then the author uses Inverse Systems to compute the postulations of n-th fat points in some cases, and the dimension of some secant varieties of varieties parameterizing forms of the type $L^{d-j}F$ where $L$ is a linear form in 3 variables and $F$ is a form of degree $j$ in 3 variables.*
- 2001-2005. PhD in Mathematics at Università degli Studi di Milano. Thesis title: *"Varieties parameterizing forms and their secant varieties"*. Advisor: professor Alessandro Gimigliano (University of Bologna).

*In this thesis the author study various problems related with varieties parameterizing forms or tensors:*

- *Secant varieties to osculating varieties to Veronese varieties (the dimensions are computed in many cases);*
- *Secant varieties to varieties that parameterize forms that can be written ad the product of forms in different degrees (the dimensions are computed in some cases);*
- *Varieties that parameterize forms in $n+1$-variables that can be written as the product of $d$ linear forms (the dimension of their secant varieties are computed in many cases; moreover it is found a contraexample to a conjecture formulated by R. Ehrenborg in 1999 that states that the secant varietis to the Grassmannian $G(n-1, n+d-1)$ is the same of the one of secant varieties of those varieties parameterizing forms in $n+1$-variables that can be written as the product of $d$ linear forms);*
- *Secant varieties of Segre varieties (the author relates two different approaches for studying those varieties: the one of Inverse System and a new one in Representation Theory).*

Some parts of this thesis where produced during missions abroad (see 1. and 2. in secation "6 Missions abroad" of this Curriculum).

# 3 Employment

- 2001–2005 PhD in Mathematics at Università degli Studi di Milano.
- September 2005–January 2006, Research Assistant, Texas A&M University, College Station, Texas, USA.
- June 28–September 27, 2006, Investigadore extranjero en la UCM (Universidad Complutense de Madrid, Spagna), financed by "GRUPO SANTANDER".
- November 2005–November 2006, "Assegnista di ricerca" at Alma Mater Studiorum Università degli studi Bologna, Italy.
- November 2006–November 2007, "Assegnista di ricerca" at Alma Mater Studiorum Università degli studi Bologna, Italy.
- November 2007–November 2009, "Assegnista di ricerca" at Alma Mater Studiorum Università degli studi Bologna, Italy.
- July 2008, Teaching Assistant at MSRI - Mathematical Sciences Research Institute - (Berkeley, California - USA).
- November 2, 2009–November 1, 2010: Post Doc at CIRM (Centro Internazionale per la Ricerca Matematica) - Fondazione Bruno Kessler (Trento - Italy).
- Janary 17 - February 26, 2011, **Visitor** at Mittag-Leffler Intitut (The Royal Swedish academy of sciences) invited da A. Dickenstein, S. Di Rocco, R. Piene, K. Ranestad e B. Sturmfels for the Spring Semester 2011 "Algebraic geometry with a view towards applications".
- November 8, 2010–November 7, 2012: Marie Curie Fellow (Post Doc) at INRIA (Institut National de Recherche en Informatique et en Automatique) Sophia Antipolis - Méditerranée (Francia).

# 4 Papers e Preprints

## 4.1 PhD Thesis

0. "Varieties parameterizing forms and their secant varieties"
(Settore Scientifico Disciplinare MAT/03).
Alessandra Bernardi
Relatore: Professor Alessandro Gimigliano (Univ. di Bologna). il Dipartimento di Matematica "Federigo Enriques" dell' Università degli Studi di Milano, titolo conseguito il 13 Febbraio 2006.

The *ABSTRACT* is at Section 2 of this Curriculum.

## 4.2 Published

1. "Varietà che parametrizzano forme e loro varietà delle secanti"
   A. Bernardi.
   Bollettino U.M.I. La Matematica nella Società nella Cultura. Serie VIII, Vol. X–A, Agosto 2007, 191–194.

   The *ABSTRACT* is at Section 2 of this Curriculum.

2. "On generalized Kummer of rank-3 vector bundles over a genus 2 curve"
   A. Bernardi, D. Fulghesu.
   "Le Matematiche" (Catania) Vol. **LVIII** (2003) - Fasc. II pp. 237–255 (2005). MR2216133 (2007b:14075).

   DOI: not assigned

   *Let $X$ be a smooth projective complex curve and let $U_X(r,d)$ be the moduli space of semi-stable vector bundles of rank $r$ and degree $d$ on $X$. It contains an open Zariski subset $U_X(r,d)^s$ which is the coarse moduli space of stable bundles, i.e. vector bundles satisfying inequality $\frac{d_F}{r_F} < \frac{d_E}{r_E}$. The complement $U_X(r,d) \setminus U_X(r,d)^s$ parametrizes certain equivalence classes of strictly semi-stable vector bundles which satisfy the equality $\frac{d_F}{r_F} = \frac{d_E}{r_E}$. Each equivalence class contains a unique representative isomorphic to the direct sum of stable bundles. Furthermore one considers subvarieties $\mathrm{SU}_X(r,L) \subset U_X(r,d)$ of vector bundle of rank $r$ with determinant isomorphic to a fixed line bundle $L$ of degree $d$. In this work we study the variety of strictly semi-stable bundles in $\mathrm{SU}_X(3,\mathcal{O}_X)$, where $X$ is a genus 2 curve. We call this variety the generalized Kummer variety of $X$ and denote it by $\mathrm{Kum}_3(X)$. Recall that the classical Kummer variety of $X$ is defined as the quotient of the Jacobian variety $\mathrm{Jac}(X) = U_X(1,0)$ by the involution $L \mapsto L^{-1}$. It turns out that our $\mathrm{Kum}_3(X)$ has a similar description as a quotient of $\mathrm{Jac}(X) \times \mathrm{Jac}(X)$ which justifies the name. We will see that the first definition allows one to define a natural embedding of $\mathrm{Kum}_3(X)$ in a projective space. The second approach is useful in order to give local description of $\mathrm{Kum}_3(X)$.*

3. "Some defective secant varieties to osculating varieties of Veronese surfaces"
   A. Bernardi, M. V. Catalisano.
   "Collect. Math." **57** (2006), no. 1, pp. 43–68. MR2206180 (2007d:14096).
   DOI: not assigned

   *We consider the $k$-osculating varieties $O_{k,d}$ to the Veronese $d-$uple embeddings of $\mathbb{P}^2$. By studying the Hilbert function of certain zero-dimensional schemes $Y \subset \mathbb{P}^2$, we find the dimension of $O^s_{k,d}$, the $(s-1)^{th}$ secant varieties of $O_{k,d}$, for $3 \leq s \leq 6$ and $s = 9$, and we determine whether those secant varieties are defective or not.*

4. "Osculating varieties of Veronese Varieties and their higher secant varieties"
   A. Bernardi, M.V. Catalisano, A. Gimigliano e M. Idà.
   "Canadian Journal of Math" Vol. **59** (3), 2007 pp. 488–502. MR2319156 (2008g:14095).
   DOI: not assigned

   *We consider the $k$-osculating varieties $O_{k,n.d}$ to the (Veronese) $d-$uple embeddings of $\mathbb{P}^n$. We study the dimension of their higher secant varieties via inverse systems (apolarity). By associating certain 0-dimensional schemes $Y \subset \mathbb{P}^n$ to $O^s_{k,n,d}$ and by studying their Hilbert function we are able, in several cases, to determine whether those secant varieties are defective or not.*

5. "Ideals of varieties parameterized by certain symmetric tensors".
   A. Bernardi
   "Journ. of P. and A. Algebra" Vol. **212** (6), 2008 pp. 1542–1559. MR2391665 (2009c:14106).
   DOI information: 10.1016/j.jpaa.2007.10.022

   *The ideal of a Segre variety $\mathbb{P}^{n_1} \times \cdots \times \mathbb{P}^{n_t} \hookrightarrow \mathbb{P}^{(n_1+1)\cdots(n_t+1)-1}$ is generated by the 2-minors of a generic hypermatrix of indeterminates. We extend this result to the case of Segre-Veronese varieties. The main tool is the concept of "weak generic hypermatrix" which allows us to treat also the case of projection of Veronese surfaces from a set of general points and of Veronese varieties from a Cohen-Macaulay subvariety of codimension 2.*

6. "Secant varieties to osculating varieties of Veronese embeddings of $\mathbb{P}^n$."
   A. Bernardi, M.V. Catalisano, A. Gimigliano e M. Idà.
   "Journal of Algebra" **321** (2009) pp. 982–1004. MR2488563 (2010d:14073).
   DOI information: 10.1016/j.algebra.2008.10.020

   *A well known theorem by Alexander-Hirschowitz states that all the higher secant varieties of $V_{n,d}$ (the d-uple embedding of $\mathbb{P}^n$) have the expected dimension, with few known exceptions. We study here the same problem for $T_{n,d}$, the tangential variety to $V_{n,d}$, and prove a conjecture, which is the analogous of Alexander-Hirschowitz theorem, for $n \leq 9$. Moreover. we prove that it holds for any $n, d$ if it holds for $d = 3$. Then we generalize to the case of $O_{k,n,d}$, the k-osculating variety to $V_{n,d}$, proving, for $n = 2$, a conjecture that relates the defectivity of $\sigma_s(O_{k,n,d})$ to the Hilbert function of certain sets of fat points in $\mathbb{P}^n$.*

7. "On the variety parametrizing completely decomposable polynomials."
   E. Arrondo, A. Bernardi. "Journal of Pure and Applied Algebra" **215** (2011) pp. 201–220.
   DOI information: 10.1016/j.jpaa.2010.04.008

   *The purpose of this paper is to relate the variety parameterizing completely decomposable homogeneous polynomials of degree $d$ in $n+1$ variables on an algebraically closed field, called $\mathrm{Split}_d(\mathbb{P}^n)$, with the Grassmannian of $n-1$ dimensional projective subspaces of $\mathbb{P}^{n+d-1}$. We compute the dimension of some secant varieties to $\mathrm{Split}_d(\mathbb{P}^n)$ and find a counterexample to a conjecture that wanted its dimension related to the one of the secant variety to $\mathbb{G}(n-1, n+d-1)$. Moreover by using an invariant embedding of the Veronse variety into the Plücker space, then we are able to compute the intersection of $\mathbb{G}(n-1, n+d-1)$ with $\mathrm{Split}_d(\mathbb{P}^n)$, some of its secant variety, the tangential variety and the second osculating space to the Veronese variety.*

8. "Computing symmetric rank for symmetric tensors."
   A. Bernardi, A. Gimigliano, M. Idà.
   "Journal of Symbolic Computation" **46** (2011) 34–53.
   DOI information: 10.1016/j.jsc.2010.08.001

   *We consider the problem of determining the symmetric tensor rank for symmetric tensors with an algebraic geometry approach. We give algorithms for computing the symmetric rank for $2 \times ... \times 2$ tensors and for tensors of small border rank. From a geometric point of view, we describe the symmetric rank strata for some secant varieties of Veronese varieties.*

9. "On the $X$-rank with respect to linear projections of projective varieties."
   E. Ballico, A. Bernardi.
   Mathematische Nachrichten. In Press.

   *In this paper we improve the known bound for the $X$-rank $R_X(P)$ of an element $P \in \mathbb{P}^N$ in the case in which $X \subset \mathbb{P}^n$ is a projective variety obtained as a linear projection from a general $v$-dimensional subspace $V \subset \mathbb{P}^{n+v}$. Then, if $X \subset \mathbb{P}^n$ is a curve obtained from a projection of a rational normal curve $C \subset \mathbb{P}^{n+1}$ from a point $O \subset \mathbb{P}^{n+1}$, we are able to describe the precise value of the $X$-rank for those points $P \in \mathbb{P}^n$ such that $R_X(P) \leq R_C(O) - 1$ and to improve the general result. Moreover we give a stratification, via the $X$-rank, of the osculating spaces to projective cuspidal projective curves $X$. Finally we give a description and a new bound of the $X$-rank of subspaces both in the general case and with respect to integral non-degenerate projective curves.*

10. "Higher secant varieties of $\mathbb{P}^n \times \mathbb{P}^m$ embedded in bi-degree $(1, d)$."
    A. Bernardi, E. Carlini, M. V. Catalisano.
    Accepted for the publication by Journal of Pure and Applied Algebra.

    *Let $X^{(n,m)}_{(1,d)}$ denote the Segre-Veronese embedding of $\mathbb{P}^n \times \mathbb{P}^m$ via the sections of the sheaf $\mathcal{O}(1, d)$. We study the dimensions of higher secant varieties of $X^{(n,m)}_{(1,d)}$ and we prove that there is no defective $s^{th}$ secant variety, except possibly for $n$ values of $s$. Moreover when $\binom{m+d}{d}$ is multiple of $(m+n+1)$, the $s^{th}$ secant variety of $X^{(n,m)}_{(1,d)}$ has the expected dimension for every $s$.*

## 4.3 Preprints

11. "On the $X$-rank with respect to linearly normal curves."
E. Ballico, A. Bernardi.
Preprint: `http://arxiv.org/abs/1002.1578`

*In this paper we study the $X$-rank of points with respect to smooth linearly normal curves $X \subset \mathbb{P}^n$ of genus $g$ and degree $n+g$.*
*We prove that, for such a curve $X$, under certain circumstances, the $X$-rank of a general point of $X$-border rank equal to $s$ is less or equal than $n+1-s$.*
*In the particular case of $g=2$ we give a complete description of the $X$-rank if $n=3,4$; while if $n \geq 5$ we study the $X$-rank of points belonging to the tangential variety of $X$.*

12. "Decomposition of homogeneous polynomials with low rank."
E. Ballico, A. Bernardi.
Preprin: `http://arxiv.org/abs/1003.5157`

*Let $F$ be a homogeneous polynomial of degree $d$ in $m+1$ variables defined over an algebraically closed field of characteristic zero and suppose that $F$ belongs to the $s$-th secant varieties of the standard Veronese variety $X_{m,d} \subset \mathbb{P}^{\binom{m+d}{d}-1}$ but that its minimal decomposition as a sum of $d$-th powers of linear forms $M_1, \dots, M_r$ is $F = M_1^d + \cdots + M_r^d$ with $r > s$. We show that if $s + r \leq 2d+1$ then such a decomposition of $F$ can be split in two parts: one of them is made by linear forms that can be written using only two variables, the other part is uniquely determined once one has fixed the first part. We also obtain a uniqueness theorem for the minimal decomposition of $F$ if the rank is at most $d$ and a mild condition is satisfied.*

13. "Stratification of the fourth secant variety of Veronese variety via the symmetric rank."
E. Ballico, A. Bernardi.
Preprint: `http://arxiv.org/abs/1005.3465`

*If $X \subset \mathbb{P}^n$ is a projective non degenerate variety, the $X$-rank of a point $P \in \mathbb{P}^n$ is defined to be the minimum integer $r$ such that $P$ belongs to the span of $r$ points of $X$. We describe the complete stratification of the fourth secant variety of any Veronese variety $X$ via the $X$-rank. This result has an equivalent translation in terms both of symmetric tensors and homogeneous polynomials. It allows to classify all the possible integers $r$ that can occur in the minimal decomposition of either a symmetric tensor or a homogeneous polynomials of $X$-border rank 4 as a linear combination of either completely decomposable tensors or powers of linear forms respectively.*

14. "Minimal decomposition of binary forms with respect to tangential projections."
E. Ballico, A. Bernardi.
Preprint: `http://arxiv.org/abs/1007.2822`

*Let $C \subset \mathbb{P}^n$ be a rational normal curve and let $\ell_O : \mathbb{P}^{n+1} \dashrightarrow \mathbb{P}^n$ be any tangential projection form a point $O \in T_A C$ where $A \in C$. In this paper we relate the minimum number $r$ of addenda that are needed to write a binary form $p$ of degree $(n+1)$ and defined over an algebraically closed field of characteristic zero as linear combination of $(n+1)$-th powers of linear binary forms $L_1, \dots, L_r$, with the minimum number of addenda that are required to write $\ell_O(p)$ as linear combination of elements belonging to $\ell_O(C)$.*

15. "Partial stratification of secant varieties of Veronese varieties via curvilinear subschemes"
E. Ballico, A. Bernardi.
Preprint: `http://arxiv.org/abs/1010.3546`

*We give a partial stratification of the secant varieties of the order $d$ Veronese variety of $\mathbb{P}^m$. We will focus on points lying on the span of curvilinear subschemes of Veronese varieties and we compute their symmetric rank for small border rank. We will also describe the structure of the Hilbert schemes of curvilinear subschemes of Veronese varieties.*

16. "Multihomogeneous polynomial decomposition using moment matrices"
A. Bernardi, J. Brachart, P. Comon, B. Mourrain.
Preprint. Submitted to ISAAC-11

*In the paper, we address the important problem of tensor decompositions which can be seen as a generalisation of Singular Value Decomposition for matrices. We consider general multilinear and multihomogeneous tensors. We show how to reduce the problem to a truncated moment matrix problem and give a new criterion for flat extension of Quasi-Hankel matrices. We connect this criterion to the commutation characterisation of border bases. A new algorithm is described which applies for general multihomogeneous tensors, extending the approach of J.J. Sylvester on binary forms. An example illustrates the algebraic operations involved in this approach and how the decomposition can be recovered from eigenvector computation.*

17. "Symmetric tensor rank with a tangent vector: a generic uniqueness theorem".
E. Ballico, A. Bernardi.
Preprint: `http://arxiv.org/abs/1101.5090`

*Let $X_{m,d} \subset \mathbb{P}^N$, $N := \binom{m+d}{m} - 1$, be the order $d$ Veronese embedding of $\mathbb{P}^m$. Let $\tau(X_{m,d}) \subset \mathbb{P}^N$, be the tangent developable of $X_{m,d}$. For each integer $t \geq 2$ let $\tau(X_{m,d}, t) \subseteq \mathbb{P}^N$, be the joint of $\tau(X_{m,d})$ and $t-2$ copies of $X_{m,d}$. Here we prove that if $m \geq 2$, $d \geq 7$ and $t \leq 1 + \lfloor \binom{m+d-2}{m}/(m+1) \rfloor$, then for a general $P \in \tau(X_{m,d}, t)$ there are uniquely determined $P_1, \dots, P_{t-2} \in X_{m,d}$ and a unique tangent vector $\nu$ of $X_{m,d}$ such that $P$ is in the linear span of $\nu \cup \{P_1, \dots, P_{t-2}\}$, i.e. a degree $d$ linear form $f$ associated to $P$ may be written as*

$$f = L_{t-1}^{d-1} L_t + \sum_{i=1}^{t-2} L_i^d$$

*with $L_i$, $1 \leq i \leq t$, uniquely determined (up to a constant) linear forms on $\mathbb{P}^m$.*

18. "Higher secant varieties of $\mathbb{P}^n \times \mathbb{P}^1$ embedded in bi-degree $(a, b)$".
E. Ballico, A. Bernardi, M. V. Catalisano.
Preprint: `http://arxiv.org/abs/1103.1151`.

*In this paper we compute the dimension of all the higher secant varieties to the Segre-Veronese embedding of $\mathbb{P}^n \times \mathbb{P}^1$ via the section of the sheaf $\mathcal{O}(a, b)$ for any $n, a, b \in \mathbb{Z}^+$. We relate this result to the Grassmann Defectivity of Veronese varieties and we classify all the Grassmann $(1, s-1)$-defective Veronese varieties.*

# 5 Research Interests

Alessandra Bernardi's interests of research are in the field of Algebraic Geometry. In particular:

- Varieties parameterizing forms,
- Varieties parameterizing certain kind of tensors both in the complex case and in the real case,
- Rank of symmetric tensors and structured tensors,
- $X$-rank of points with respect to projective varieties $X$,
- Uniqueness of the decomposition of a tensor in terms of tensors of rank 1,
- "Adjoint varieties" (homogeneous varieties associated to Lie algebras),
- Secant varieties,
- Zero-dimensional schemes and their postulations,
- Grassmannians.

## 5.1 Past research interests

On what concerns the past research activity see sections 2 and 4.

## 5.2 Present research interests

In the present Alessandra Bernardi is working on the following argouments:

1. Generation of algorithms for the computation of the rank of a symmetric tensor both in the real case (this is in collaboration with professor G. Ottaviani of the Università di Firenze–Italy) and in the complex case (this is in collaboration with professor E. Ballico of the Università di Trento–Italy).
2. Dimension of the secant varieties to Segre-Veronese varieties (in collaboration with professor M. V. Catalisano of the Università di Genova–Italy and with professor E. Ballico of the Università di Trento–Italy).
3. Dimension of secant varieties of osculating varieties of Veronese varieties.
4. Conjecture on rank and symmetric rank of a symmetric tensor.

## 5.3 Projects for the future

- Marie Curie Fellow at INRIA Sophia Antipolis - Méditerranée - Nice (France).
  Scientific in charge: Prof. Bernard Mourrain.
  Project: FP7-PEOPLE-2009-IEF - 252367 - DECONSTRUCT: "Decomposition of Structured Tensors, Algorithms and Characterization".
  Beginning of the project: November first, 2010.
  Duaration: 2 years.

  ABSTRACT: Tensors play a wide role in numerous application areas as Signal Processing for Telecommunications, Arithmetic Complexity or Data Analysis. In some applications tensors may be completely symmetric, or symmetric only in some modes, or may not be symmetric. In most of these applications, the decomposition of a tensor into a sum of rank-1 terms is relevant, since tensors of interest have a reduced rank. Most of them are structured i.e. they are either symmetric or enjoy some index-invariance. Lastly, they are often real, which raises open problems concerning the existence and calculation of the decompositions. These issues build the basic bricks of the research program we propose. The classes of tensors described above have a geometric translations in terms of classical algebraic varieties: Segre, Veronese, Segre-Veronese varieties and Grassmannians and their secant varieties. A complete description of equations for those secant varieties and their dimensions is still not known (only dimensions of secant varieties to Veronsean are classified), although they have been studied by algebraic and differential geometers and algebraists for a long period up to now. The aim of this research project is:
  - to attack both the description of the ideal of those secant varieties and their dimensions, starting from low dimensions and low degrees,
  - to propose algorithms able to compute the rank of structured tensors.

  Workshops in Palo Alto (CA-USA, 2008) and in Nice (FR, 2009) showed that Italy and France are among the most active in Europe in the field of tensor decompositions. Both the coordinator of this project and the hosting organization have already obtained results in this field regarding equations and algorithms. Hence this program is crucial for the development of those research areas in the European Community, along with the numerous international collaborations already existing. The impact of this project will be visible in both academic and industrial worlds.

## 5.4 Research projects for the future

The future projects are:

- to continue the study of secant varieties to Segre-Veronese varieties with professor M. V. Catalisano (Università di Genova), and professor E. Ballico (Università di Trento).

- To continue a collaboration with professor E. Ballico (Univ. di Trento) on:
  - the computation of the $X$-rank with respect to a projective variety $X$,
  - the uniqueness of the decomposition of symmetric tensors via completely decomposable tensors,
  - the stratification of the secant varieties to Veronese varieties via symmetric rank.
- To continue the collaboration with prof. G. Ottaviani (Univ. di Firenze) on the computation of the real rank of homogeneous polynomials in 3 variables of degree 3 and 4.
- to continue above collaboration with M.V. Catalisano (Università di Genova), M. Idà and A. Gimigliano (Università di Bologna) on the secant varieties of varieties parameterizing forms or tensors.
- to begin a collaboration with the laboratory of INRIA - Sophie Antipolis (Nice, France) with B. Mourrain, P. Comon and their research group with the following objectives:
  1. Equations for r-th secant varieties of:
     - Veronese varieties,
     - Osculating varieties to Veronese varieties,
     - Chow variety of zero cycles,
     - Segre varieties,
     - Segre-Veronese varieties,
     - Grassmannians,
     - Flag varieties,

     (a) Give a complete analysis of small cases useful in applications (dimension and degree smaller than 5);

     (b) Generalize as much as possible to higher values of degree and dimension.
  2. Classification, via generation of Algorithms, of the above secant varieties in terms of:
     - Dimension (i.e. border rank of structured tensors),
     - rank of structured tensors.
  3. Tackle those problems when those varieties are defined over:
     - an algebraically closed field of characteristic zero (field with a more advanced and classical literature),
     - the field of real numbers (field mostly required for applications).

# 6 Missions abroad

1. September 17 - December 17, 2004, at Queen's University (Kingston, Ontario, Canada), invited by professor A.V. Geramita;

   *During this visit the research area was:*
   - *"secant varietis of osculating varietis of Veronese varieties" with professor A.V.Geramita;*
   - *representation theory techniques to study the problem of generation of the ideals of "Secant varieties to Segre varieties" with professors M. Roth and I. Dimitrov.*

   *The results obtained during this visit were posted in the PhD thesis of Alessandra Bernardi.*

2. January 9 - March 1, 2005, at Universidad Complutense de Madrid (Madrid, Spain), invited by professor E. Arrondo;

   *During this visit the research area was the study of secant varieties to varieties parameterizing forms that can be written as product of linear forms. We produced a contraexamle to the conjecture of Ehrenborg (1999) that stated that the dimension of certain secant varieties to Grassmanians was the same of certain secant varieties to varieties paramterizing forms splitting as a product of linear forms. The results obtained during this visit were posted in the PhD thesis of Alessandra Bernardi.*

3. September 26 - October 15, 2005, at Texas A&M University (College Station, Texas, USA),Researcher Assistant for the PhD course "MATH 689-computational complexity geometry" for professor J.M. Landsberg;

   *During this visit, beside the above research assistant activity, Alessandra Bernardi started the study of the dimensions of the first secant varieties to adjoint varieties (homogeneous varieties associated to Lie Algebras) with professor J. M Landsberg.*

4. October 27 - December 15, 2005, at Texas A&M University (College Station, Texas, USA),Researcher Assistant for the PhD course "MATH 689-computational complexity geometry" for professor J.M. Landsberg;

   *During this visit, beside the above research assistant activity, Alessandra Bernardi continued the study of the dimensions of first secant varieties to adjoint varieties begun in the previous visit.*

5. March 2 - April 15, 2006, at Texas A&M University (College Station, Texas, USA), research assistant for professor J.M. Landsberg.

   *During this visit, beside the above research assistant activity, Alessandra Bernardi continued the study of the dimensions of first secant varieties to adjoint varieties begun in the previous visit. The results obtained are still in preparation.*

6. June 28 - September 27, 2006, at Universidad Complutense de Madrid (Madrid, Spain), invited by professor E. Arrondo for a "Programa de visitantes distinguidos e investigadores extranjeros en la UCM" financed by "GRUPO SANTANDER".

   *During this visit, Alessandra Bernardi studied with the collaboration of professor E. Arrondo the intersection locus between Grassmannians and*

   - *Veronese varieties,*
   - *Secant varieties to Veronese varieties,*
   - *Tangential varieties and Veronese varieties,*
   - *Osculating varieties to Veronese varieties,*
   - *Varieties parameterizing forms that are decomposable as product of linear forms.*

7. July 7 - 20, 2008, at MSRI (Mathematical Sciences Research Institute), Berkeley (California - USA) invited as Teaching Assistant for the Graduate Workshop "Geometry and representation theory of tensors for computer science, statistics and other areas".

   *This workshop was organized by J.M. Landsberg (Texas A&M - Texas - USA), Lek-Heng Lim (UC Berkeley - California - USA) and Jason Morton (UC Berkeley - California - USA) with the goal of introduce relevant topics in Geometry and Representation Theory to PhD students. Computational complexity, statistical learning theory, signal processing, scientific data analysis, were recently formulated in geometric terms via representation theory. Specifically the problem studied was the one of "matrix multiplication". During the second week of the workshop was possible to work on open problems.*

8. July 21 - 27, 2008, at AIM (American Institute of Mathematics) of Paolo Alto (California - USA) for the Workshop "Geometry and representation theory of tensors for computer science, statistics and other areas".

   *This workshop was organized by J.M. Landsberg (Texas A&M - Texas - USA), Lek-Heng Lim (UC Berkeley - California - USA), Jason Morton (UC Berkeley - California - USA) and Jerzy Weyman (Northeastern University - Boston - MA - USA).*
   *It was devoted to the study of problems in quantum computing, complexity theory, statistical learning theory, signal processing, and data analysis, problems in algebraic geometry and representation theory. In all these areas there are varieties in tensor spaces that are invariant by change of coordinates.*
   *The work of this workshop was to translate problems from the applied world into mathematical language and solve them where possible.*

9. September 1 - 7, 2008, at the Mathematical department of Universidad Complutense de Madrid (Spagna), invited by professor E. Arrondo.

   *During this visit, Alessandra Bernardi studied with the collaboration of professor E. Arrondo the intersection locus between Grassmannians and*

   - *Veronese varieties,*
   - *Secant varieties to Veronese varieties,*
   - *Tangential varieties and Veronese varieties,*
   - *Osculating varieties to Veronese varieties,*
   - *Varieties parameterizing forms that are decomposable as product of linear forms.*

10. January 21 - 23 2009, at the Laboratoire d'Informatique, Signaux et Systèmes de Sophia-Antipolis (Francia), invited by professor P. Comon.

    *During this visit Alessandra Bernardi and Pierre Comon prepared a research project on*

    - *the conjecture on the possible equivalence between the symmetric rank and the rank itself of a symmetric tensor;*
    - *how many generic ranks there exist in the case of real tensor (there is a conjecture that states that they are actually two);*
    - *the closure of the set of tensors of rank less or equal to a fixed integer.*

11. February 1 - 5 2009, at the math department of Universidad Complutense de Madrid (Spagna),invited by professor E. Arrondo.

    *During this visit they finished the study of the intersection locus between Grassmannians and:*

    - *Varieties parameterizing forms that can be written ad product of linear forms,*
    - *Some of their secant varieties,*
    - *Veronese varieties,*
    - *Tangential varieties to Veronese varieties,*
    - *Second osculating spaces to Veronese varieties,*
    - *Some secant varieties to Veronese varieties.*

# 7 Teaching activity abroad

- Researcher Assistant for the American PhD course in Math.: **MATH 689-computational complexity geometry**, I Semester A.A. 2005-2006, Texas A&M University, College Station (Texas, USA); teaching professor: J. M. Landsberg (College Station, Texas, USA).
- Teaching Assistant for professors J.M. Landsberg (Texas A&M - Texas - USA), Lek-Heng Lim (UC Berkeley - California - USA) and Jason Morton (UC Berkeley - California - USA) during the **Graduate Workshop "Geometry and representation theory of tensors for computer science, statistics and other areas"**, at MSRI (Mathematical Sciences Research Institute), Berkeley (California - USA). July 2008.

# 8 Italian teaching activity

1. **Geometria e Algebra I**, I year course, Tutorial exercises, I semester, A.A. 2002-2003 teaching professor G. Bolondi, Faculty of Ingegneria Matematica e Fisica, Politecnico di Milano.
2. **Elementi di Analisi Matematica, Algebra e Geometria**. I year course, Tutorial exercises, I semester, A.A. 2003-2004, teaching professor F. Colombo, faculty of Ingegneria Meccanica, Politecnico di Milano.
3. **Geometria e Algebra Lineare, (allievi A-K)**. I year course, Tutorial exercises, I semester, A.A. 2003-2004 teaching professor A. Gimigliano, faculty of Ingegneria Gestionale, Università di Bologna.

4. **Geometria e Algebra Lineare, (allievi L-Z)**. I year course, Tutorial exercises, I semester, A.A. 2003-2004 teaching professor A. Gimigliano, faculty of Ingegneria Gestionale, Università di Bologna.
5. **Analisi Matematica B (allievi Ingegneri Civili)**. I year course, Tutorial exercises, II semester, A. A. 2004-2005, Faculty of Ingegneria Civile, Milano Leonardo Politecnico; teaching professor G. Verzini.
6. **Geometria e Algebra Lineare, (allievi A-K)**. I year course, Tutorial exercises, I semester, A.A. 2006-2007teaching professor A. Gimigliano, faculty of Ingegneria Gestionale, Università di Bologna.
7. **Geometria e Algebra Lineare, (allievi L-Z)**. I year course, Tutorial exercises, I semester, A.A. 2006-2007 teaching professor A. Gimigliano, faculty of Ingegneria Gestionale, Università di Bologna.
8. **Geometria e Algebra Lineare, (allievi A-K, L-Z)**. I year course, Tutorial exercises, I semester, A.A. 2007-2008 teaching professor A. Gimigliano, faculty of Ingegneria Gestionale, Università di Bologna.
9. **Geometria e Algebra (Corso integrato, moduli di Algebra e Geometria)**, II semester, A.A. 2007-2008, teaching professor A. Gimigliano, C.d.L. Scienza della Formazione Primaria, faculty of Scienza della Formazione, Università degli studi di Bologna.
10. **Analisi Matematica I, Geometria e Algebra Lineare (Corso integrato, moduli di Geometria e Algebra Lineare), (allievi A-K)**.I year course, Tutorial exercises, I semester, A.A. 2008-2009 teaching professor A. Gimigliano, faculty of Ingegneria Gestionale, Università di Bologna.
11. **Analisi Matematica I, Geometria e Algebra Lineare (Corso integrato, moduli di Geometria e Algebra Lineare), (allievi A-K)**.I year course, Tutorial exercises, I semester, A.A. 2009-2010 teaching professor A. Gimigliano, faculty of Ingegneria Gestionale, Università di Bologna.
12. **Analisi Matematica I, Geometria e Algebra Lineare (Corso integrato, moduli di Geometria e Algebra Lineare), (allievi L-Z)**.I year course, Tutorial exercises, I semester, A.A. 2009-2010 teaching professor A. Gimigliano, faculty of Ingegneria Gestionale, Università di Bologna.

# 9 Research Projects

## 9.1 Research Project won by Alessandra Bernardi

- **Title of the project**: "Programa de visitantes distinguidos e investigadores extranjeros en la UCM".
  **Financed by**: GRUPO SANTANDER.
  **Duration**: 3 months; **period**: 28 Giugno - 27 Settembre 2006.
  **Responsible**: Prof. Enrique Arrondo (Universidad Complutense de Madrid - Spagna).
- **Title of the project**: "Dimensione e ideali di varietà delle secanti di varietà che parametrizzano forme e/o tensori. Generazione di algoritmi per il computo del rango strutturato dei loro elementi."
  **Financed by**: CIRM – FBK (Trento).
  **Duration**: 1 year; **period**: November 2009 – November 2010.
  **Responsible**: Prof. Edoardo Ballico (Università degli Studi di Trento).
- **Title of the project**: "Decomposition of Structured Tensors, Algorithms and Characterization".
  **Financed by**: Comunità Europea nell'ambito del programma FP7-PEOPLE-2009-IEF - 252367, Marie Curie Fellow.
  **Duration**: 2 anni; **period**: Novembre 2010 – Novembre 2012.
  **Responsible**: Prof. Bernard Mourrain (INRIA Sophia Antipolis - Méditerranée - Nice -France)

## 9.2 Additional Research Projects

- **Title of the project**: "Questioni di Geometria, Topologia e Algebra".
  **Financed by**: Università degli Studi di Milano, Dipartimento di Matematica "Federigo Enriques".
  **Duration**: 1 year; **period**: 2002.
  **Responsible**: Prof. Antonio Lanteri (Università degli Studi di Milano).

- **Title of the project** "Questioni di Geometria, Topologia e Algebra".
  **Financed by**: Università degli Studi di Milano, Dipartimento di Matematica "Federigo Enriques".
  **Duration**: 1 year; **period**: 2003.
  **Responsible**: Prof. Antonio Lanteri (Università degli Studi di Milano).

- **Title of the project**: "Questioni di Geometria, Topologia e Algebra".
  **Financed by**: Università degli Studi di Milano, Dipartimento di Matematica "Federigo Enriques".
  **Duration**: 1 year; **period**: 2004.
  **Responsible**: Prof. Antonio Lanteri (Università degli Studi di Milano).

- **Title of the project**: "Geometria sulle varietà algebriche".
  **Financed by**: MIUR (Ministry of Education and Research of the Italian Government) + Università degli Studi di Milano.
  **Duration**: 2 years; **period**: 2002-2004.
  **Responsibles**: Prof. Antonio Lanteri (Università degli studi di Milano), Prof. Alessandro Verra (Università di Roma III).

- **Title of the project**: "Questioni di Geometria, Topologia e Algebra".
  **Financed by**: Università degli Studi di Milano, Dipartimento di Matematica "Federigo Enriques".
  **Duration**: 1 year; **period**: 2005.
  **Responsible**: Prof. Antonio Lanteri (Università degli Studi di Milano).

- **Title of the project**: "Geometria sulle varietà algebriche".
  **Financed by**: MIUR (Ministry of Education and Research of the Italian Government) e Università degli Studi di Milano.
  **Duration**: 2 years; **period**: 2005-2006.
  **Responsibles**: Prof. Lambertus Van Germen (Univerisità degli Studi di Milano), Prof. Alessandro Verra (Università di Roma III).

- **Title of the project**: "Project PRIN 2004 (Progetti di rilevante interesse nazionale)".
  **Financed by**: National government funds.
  **Duration**: 2 years; **period**: 2004-2005.
  **Responsible**: Prof. Angelo Vistoli (Università degli Studi di Bologna).

- **Title of the project**: "RFO 2006 funds (Ricerca Fondamentale Orientata)."
  **Financed by**: Università degli studi di Bologna.
  **Duration**: 1 year; **period**: 2006.
  **Responsible**: Prof. Mirella Manaresi (Università degli Studi di Bologna).

- **Title of the project**: "Project PRIN 2006 (Progetti di rilevante interesse nazionale)".
  **Financed by**: National government funds.
  **Duration**: 2 years; **period**: 2006-2007.
  **Responsible**: Prof. Mirella Manaresi (Università degli Studi di Bologna).

- **Title of the project**: "RFO 2007 funds (Ricerca Fondamentale Orientata)."
  **Financed by**: Università degli studi di Bologna.
  **Duration**: 1 year; **period**: 2007.
  **Responsible**: Prof. Mirella Manaresi (Università degli Studi di Bologna).

- **Title of the project**: "RFO 2008 funds (Ricerca Fondamentale Orientata)."
  **Financed by**: Università degli studi di Bologna.
  **Duration**: 1 year; **period**: 2008.
  **Responsible**: Prof. Mirella Manaresi (Università degli Studi di Bologna).

- **Title of the project**: "RFO 2009 funds (Ricerca Fondamentale Orientata)."
  **Financed by**: Università degli studi di Bologna.
  **Duration**: 1 year; **period**: 2009.
  **Responsible**: Prof. Mirella Manaresi (Università degli Studi di Bologna).

- **Title of the project**: "Project PRIN 2008 (Progetti di rilevante interesse nazionale)".
  **Financed by**: National government funds.
  **Duration**: 2 years; **period**: 2008-2009.
  **Responsible**: Prof. Mirella Manaresi (Università degli Studi di Bologna).

# 10 Invited Talks

- "Osculating varieties of Veronesean and their higher secant varieties", December 10, 2004 - CMS 2004 Winter Meeting, Montreal (Quebec, Canada);
- "Secant varieties to osculating varieties of Veronesean" , February 18, 2005 - Departamento de Álgebra, Universidad Complutense de Madrid. (Madrid, Spagna);
- "Secant varieties and Big Waring Problem", October 7, 2005, Mathematical Department, Texas A&M University (College Station,Texas, USA);
- "Varietà delle secanti a varietà che parametrizzano forme ottenute come prodotto di forme lineari", May 29, 2006, Giornate di Geometria Algebrica e argomenti correlati VIII, Dipartimento di Matematica, Università di Trieste.
- "Secant Varieties and Ideals of varieties parameterizing certain symmetric tensors", July 17, 2008, MSRI (Mathematical Sciences Research Institute) (Berkeley, California, USA);
- "Secant varieties to osculating varieties of Veronese Varieties" , September 4, 2008 - Departamento de Álgebra, Universidad Complutense de Madrid, (Madrid, Spagna);
- "Rappresentazione di varietà algebriche", October 28, 2008, Dipartimento di Matematica, Università degli studi di Bologna.
- "Varietà che parametrizzano polinomi completamente decomponibili", March 13, 2009, Dipartimento di Matematica, Università degli Studi di Firenze.
- "Sylvester's Algorithm", June 10, 2009 - Workshop on tensors and interpolation, Nice, France.
- "Dal problema di Waring alle telecomunicazioni", December 10, 2009, Università degli studi di Trento, Italy.
- "From the Waring problem to tensor rank through secant varieties", March 18, 2010, SAGA Winter School, Auron, Nice, France.
- "Dal problema di Waring alle telecomunicazioni", April 20, 2010, Università degli studi di Ancona (Italy).
- "Un assaggio di scienza nell'iconografia russa", June 17, 2010, Università degli studi di Trento (Italy).
- "Decomposition of Homogeneous Polynomials", September 15, 2010, Workshop on Tensor Decompositions and Applications (TDA 2010). September 13–17 2010. Monopoli, Bari, Italy.
- "Varietà delle secanti a varietà che parametrizzano tensori: attualità del problema di Waring, aspetti geometrici correlati ed applicazioni", October 7, 2010. Trieste, Italy.
- "Applicazioni recenti di risultati classici su variet delle secanti a variet che parametrizzano tensori. Dal problema di Waring al rango di tensori" , November 22, 2010, "Progressi Recenti in Geometria Reale e Complessa", Levico Terme (Trento, Italy), October 17–22, 2010.
- "Secant varieties and Rank of tensors", February 1, 2011, Mittag-Leffler Institute, Spring Semester: "Algebraic Geometry with a view towards applications" 17 Janaury – 15 June 2011.

# 11 Presentations

- "Sulle funzioni convesse", February 27, 2002, Dipartimento di Matematica, Università di Trieste;
- "Dimostrazione del teorema di Darboux", September 27, 2002, Dipartimento di Matematica, Università di Trieste;
- "Programma di Sarkisov in dimensione 2 per la classificazione degli Spazi Fibrati di Mori secondo la Teoria di Mori", July 18, 2002, Dipartimento di Matematica, Universtà di Milano;
- "Esposizione dell'articolo di G. Canuto *Curve associate e Formule di Pluker nelle Grassmaniane*, apparso su "Inventiones Mathematicae", 53, 77-90 (1979)", January 15, 2003, Dipartimento di Matematica, Università di Milano;
- "How one's can calculate all the differential invariants of $Seg(\mathbb{P}^n \times \mathbb{P}^n) \cap H$, where $H$ is a generic hyperplane. Understand this as a homogeneous variety of $Sl_{n+1}\mathbb{C}$", February 13, 2003, Dipartimento di Matematica, Università di Trieste;
- "Un'introduzione al problema dello studio della Postulazione dei Punti Grassi", March 19, 2003, Dipartimento di Matematica, Università degli Studi di Milano;
- "Un'introduzione al problema dello studio della Postulazione dei Punti Grassi e recenti applicazioni", May 23, 2003, Dipartimento di Matematica, Università di Pavia;
- "Waring type problems and Auxiliary varieties Associated to Veronese varieties", October 6, 2004, Mathematical Department, Queen's University (Kingston, Ontario, Canada);
- "Secant varieties to the Osculating varieties of the Veronesean", October 13, 2004, Mathematical Department, Queen's University (Kingston, Ontario, Canada);
- "Varietà delle secanti alle Veronese e applicazioni algebriche", January 26, 2005 - Departamento de Álgebra Universidad Complutense de Madrid (Madrid, Spagna);
- "Varietà delle secanti alle varietà tangenziali ed osculanti a varietà di Veronese" ,February 2, 2005- Departamento de Álgebra, Universidad Complutense de Madrid (Madrid, Spagna); (Madrid, Spagna);
- "Construction of Cominuscule Varieties" , October 6, 2005, Mathematical Department, Texas A&M University (College Station, Texas, USA);
- "An introduction to Representation Theory", November 2, 2005, Mathematical Department, Texas A&M University (College Station, Texas, USA);
- "An introduction to de Rham Cohomology", November 17, 2005, Mathematical Department, Texas A&M University (College Station, Texas, USA);
- "An introduction to de Rham Cohomology", November 18, 2005, Mathematical Department, Texas A&M University (College Station, Texas, USA);
- "An introduction to de Rham Cohomology", November 22, 2005, Mathematical Department, Texas A&M University (College Station, Texas, USA);
- "On Alexander-Hirshowitz theorem via Lemma d'Horace", December 1, 2005, Mathematical Department, Texas A&M University (College Station, Texas, USA);
- "Lemma d'Horace differentielle", December 5, 2005, Mathematical Department, Texas A&M University (College Station, Texas, USA);
- "Dall'Algebra Lineare a questioni irrisolte", May 15, 2008, Seminario per il CdL Algebra Superiore, Dipartimento di Matematica, Università di Bologna.
- "Ideale delle varietà di Segre-Veronese", June 12, 2008, DIMA UNiversità degli studi di Genova.
- "Rango e rango simmetrico di tensori simmetrici.", March 3, 2009 - Dipartimento di Matematica, Università degli Studi di Bologna.

- "Algorithms for computing the rank of a tensor", February 11, 2011, Mittag-Leffler Institute, Spring Semester: "Algebraic Geometry with a view towards applications" 17 Janaury – 15 June 2011.
- "Tenseurs", March 8, 2011, INRIA, Sophia Antipolis Mediterranée, France.

# 12 Additional Conferences and Schools Attended

- "Summer School Perugia", Perugia (Italy), July 29 - September 1, 2001;
- "Pragmatic 2003", Catania (Italy), June 9 - 28, 2003;
- "Interpolation problem and Projective embeddings", Torino (Italy), September 15 - 20, 2003;
- "Worksxhop on Algebraic curves, monodromy and related topics", Milano (Italy), April 1-2, 2004;
- "International school on Projective Geometry", Anacapri (Italy), June 1-5, 2004;
- "Projective Varieties with unexpected geometric properties", Siena (Italy), June 8-12, 2004;
- "School/Workshop on Computational Algebra for Algebraic Geometry and statistics", Torino (Italy), September 6 - 11, 2004;
- "Rt. 81 conference in honor of Graham Evans and Workshop on Resolution (for young researchers)", Cornell University of Ithaca, New York, USA, October 1-3, 2004;
- "CMS 2004 Winter meeting", Montreal, Quebec (Canada), December 10-13, 2004.
- "AGaFE, Geometry of Algebraic Varieties", Ferrara, June 22-25, 2005;
- "Texas Geometry and Topology conference", Austin, Texas, USA, September 30 - October 2, 2005;
- "Geometric and Probabilistic Methods in group theory and dynamical systems", November 4-6, 2005, Texas A&M University, College Station, Texas, USA;
- "Harvey/Polking conference, Singularities in Analysis, Geometry and Topology", November 11-13, 2005, Rice University, Houstin, Texas, USA;
- "Giornate di Geometria Algebrica e argomenti correlati VIII", Trieste, May 26-29, 2006.
- Graduate Workshop: "Geometry and representation theory of tensors for computer science, statistics and other areas", MSRI - Mathematical Sciences Research Institute - (Berkeley, California - USA) July 7 - 20 Luglio 2008.
- "Geometry and representation theory of tensors for computer science, statistics and other areas", AIM - American Institute of Mathematics - (Paolo Alto, California - USA), July 21 - 27, 2008.
- "INDAM workshop: Geometry of projective varieties" (Roma), September 30 - October 4, 2008.
- "Workshop on tensors and interpolation" June 10-11-12, 2009, INRIA-JAD-CNRS Nice, France.
- "Conference on Classical and recent aspects in the study of projective varieties. In honour of Lucian Badescu on the occasion of his 65th birthday", Genova (Italy), January 21-22, 2010.
- "SAGA Winter School", Auron, Nice, France March, 15-19, 2010.
- "INdAM Conference "Complex Geometry"", Levico Terme, Trento, Italy, May 31 - June 4, 2010.
- "Summer school: Geometry of tensors and applications", Sophus Lie Conference Center, Nordfjordeid - Norway, June 14 - 18, 2010.
- "School (and Workshop) on The Minimal Model Program and Shukurov's ACC Conjecture", Povo (Trento), 5 - 10 Giugno 2010.

- "International Conference on Perspectives on Algebraic Varieties", Levico Terme, Trento (Italy), September 5-10, 2010.
- 'Workshop on Tensor Decompositions and Applications (TDA 2010)". September 13–17 2010. Monopoli, Bari, Italy.
- "Progressi Recenti in Geometria Reale e Complessa", Levico Terme (Trento, Italy), October 17–22, 2010.
- "Algebraic Geometry in the sciences", (Oslo, Norway), January 10–14, 2011.
- Spring semester 2011 "Algebraic Geometry with a view towards applications", (Mittag-Leffler, Stoccolma, Svezia) January 17 – February 26, 2011.
- "CIAM workshop: An afternoon of biology and mathematics", KTH (Stockholm, Sweden), February 4, 2011.
- "Solving polynomial equations", KTH (Stockholm, Sweden), February 21–25, 2011.

# 13 Reviewer and Referee

## 13.1 Reviewer

Reviewer in the areas 14N05, 13D40, 14J26, 14M15, 13P10 (MSC 2000 classification) for:

- AMS Mathematical Reviews.
- Zentralblatt MATH.

## 13.2 Referee

Referee for the following journals:

- Experimental Mathematics (EM).
- International Journal of Engineering, Science and Technology (IJEST).

# 14 Reviews on Mathscinet of some of the Paper of Alessandra Bernardi

- "On generalized Kummer of rank-3 vector bundles over a genus 2 curve."
  A. Bernardi, D. Fulghesu.
  "Le Matematiche" (Catania) Vol. **LVIII** (2003) - Fasc. II pp. 237–255 (2005).

  **Review on Mathscinet MR2216133 (2007b:14075)**
  Let $X$ be a smooth projective complex curve and let $U_X(r,d)$ be the moduli space of semi-stable vector bundles of rank $r$ and degree $d$ on $X$. Denote by $SU_X(r,L)$ the subvariety of $U_X(r,d)$ determined by bundles with determinant isomorphic to $L$. In this paper the authors study the variety of strictly semi-stable bundles in $SU_X(3,O_X)$, when $X$ has genus 2. Since such a variety can be described as a quotient of $Jac(X) \times Jac(X)$ it is called the generalized Kummer variety and is denoted by $Kum_3(X)$. The Kummer variety $Kum_3(Jac(X))$ associated to $Jac(X)$ describes the completely decomposable bundles in $SU_X(3,O_X)$. The authors describe the singular locus of $Kum_3(\mathrm{Jac}(X))$ and compute its degree.
  (Reviewed by Leticia Brambila-Paz.)

- "Some defective secant varieties to osculating varieties of Veronese surfaces."
  A. Bernardi, M. V. Catalisano.
  "Collect. Math." **57** (2006), no. 1, pp. 43–68.

  **Review on Mathscinet MR2206180 (2007d:14096)**
  Let $V_d \subset \mathbb{P}^N$, $N = \binom{d+2}{2} - 1$, denote the $d$-Veronese embedding of $\mathbb{P}^2$ defined by the complete linear system of forms of degree $d$. Let $O_{k,d} \subset \mathbb{P}^N$ be the $k$th osculating variety to $V_d$, and let $O^s_{k,d} \subset \mathbb{P}^N$ be the $(s-1)$st secant variety of $O_{k,d}$.

In the paper under review, the dimension of $O^s_{k,d}$ is obtained for $3 \leq s \leq 6$ and $s = 9$ (the cases $1 \leq s \leq 2$ are already known). In particular, triples $(s, k, d)$ for which $O^s_{k,d}$ does not attain the expected dimension are determined.

The main idea of the proof is the following. One can associate to $O^s_{k,d}$ a zero-dimensional subscheme $Y \subset \mathbb{P}^2$ supported at $s$ general points. It follows by construction of $Y$ that $\dim O^s_{k,d} = N - h^0(\mathbb{P}^2, \mathcal{I}_Y(d))$. Then the postulation of $Y$ is related to the postulation of a zero-dimensional subscheme $X \subset \mathbb{P}^2$ consisting of $s$ general $(k+1)$-fat points, which is well known for $s \leq 9$.

The paper also contains some conjectures if either $7 \leq s \leq 8$ or $s$ is a square.
(Reviewed by José Carlos Sierra)

- "Osculating varieties of Veronese Varieties and their higher secant varieties."
A. Bernardi, M.V. Catalisano, A. Gimigliano e M. Idà.
"Canadian Journal of Math" Vol. **59** (3), 2007 pp. 488–502.

  **Review on Mathscinet MR2319156 (2008g:14095)**
  In this paper the authors, inspired by the successful motto "higher secant varieties of Veronese varieties are related to fat points", approach the study of higher secant varieties of osculating varieties of Veronese varieties. In this case, the key ingredients are schemes supported on points but having a non-reduced structure sitting between $k$-fat points and $(k+1)$-fat points. Following this idea the authors prove many partial results and produce a conjecture. They also relate their work to the classical problem of presenting forms as the sum of forms having a given shape. Here the summands are the power of a linear form times any form of the proper degree. Other instances of the problem have been considered [see, e.g., E. Carlini, in Projective varieties with unexpected properties, 67–79, Walter de Gruyter, Berlin, 2005; MR2202247 (2007f:14056); C. Fontanari, Ann. Univ. Ferrara Sez. VII Sci. Mat. 52 (2006), no. 1, 37–43; MR2246903 (2007g:14064)].
  (Reviewed by Enrico Carlini)

- "Ideals of varieties parameterized by certain symmetric tensors."
A. Bernardi.
"Journ. of P. and A. Algebra" Vol. **212** (6), 2008 pp. 1542–1559.

  **Review on Mathscinet MR2391665 (2009c:14106).**
  H. T. Ha and R. Grone proved that the ideal of a Segre variety is generated by all 2-minors of a generic hypermatrix of indeterminates. Bernardi considers the Segre-Veronese variety. Let $V_1, \ldots, V_t$ be vector spaces of dimensions $n_1, \ldots, n_t$ respectively. The Segre-Veronese variety $\mathcal{S}_{d_1,\ldots,d_t}(V_1 \otimes \cdots \otimes V_t)$ is the embedding of $\mathbf{P}(V_1) \otimes \cdots \otimes \mathbf{P}(V_t)$ into $\mathbf{P}^{N-1}$, where $N = \left(\prod_{i=1}^t \binom{n_i+d_i-1}{d_i}\right)$, given by sections of the sheaf $\mathcal{O}(d_1, \ldots, d_t)$. Let $\mathbf{n} = (n_1, \ldots, n_t)$ and $\mathbf{d} = (d_1, \ldots, d_t)$. The ideal of a Segre-Veronese variety is set-theoretically given by the 2-minors of an $(\mathbf{n}, \mathbf{d})$-symmetric hypermatrix of indeterminates.

  The author proves that the idea of the 2-minors of the generic $(\mathbf{n}, \mathbf{d})$-symmetric hypermatrix is the ideal of a Segre-Veronese variety. She gives the following definition:

  Let $K[u_1, \ldots, u_r]$ be a polynomial ring. A hypermatrix $\mathcal{A} = (f_{i_1,\ldots,i_t})_{1 \leq i_j \leq n_j,\, j=1,\ldots,t}$, where all $f_{i_1,\ldots,i_t} \in K[u_1, \ldots, u_r]_1$, is called a "weak generic hypermatrix of indeterminates" if:

  1. all the entries of $\mathcal{A}$ belong to $\{u_1, \ldots, u_r\}$;
  2. there exists an entry $f_{i_1,\ldots,i_t}$ such that $f_{i_1,\ldots,i_t} \neq f_{k_1,\ldots,k_t}$ for all $(k_1, \ldots, k_t) \neq (i_1, \ldots, i_t)$, $1 \leq k_j \leq n_j$, $j = 1, \ldots, t$;
  3. the ideals of 2-minors of all sections of $\mathcal{A}$ are prime ideals.

  The author proves that the ideal generated by 2-minors of a weak generic hypermatrix is a prime ideal. Then she shows that a symmetric hypermatrix of indeterminates is weak generic and she can conclude, since the ideal generated by its 2-minors defines, set-theoretically, a Segre-Veronese variety.

  Using this idea the author also studies the case of projection of Veronese surfaces from a set of general points and of Veronese varieties from a Cohen-Macaulay subvariety of codimension 2.
  (Reviewed by Giovanna Ilardi)

- "Secant varieties to osculating varieties of Veronese embeddings of $\mathbb{P}^n$."
A. Bernardi, M.V. Catalisano, A. Gimigliano e M. Idà.
"Journal of Algebra" **321** (2009) pp. 982–1004.

**Review on Mathscinet MR2488563 (2010d:14073)**

A special case of the functional analog of the classical Waring problem is the problem of finding the minimal number $\kappa(d,n)$ such that a general form (i.e., a homogeneous polynomial) of degree $d$ in $n+1$ variables can be written as a sum of $d$th powers of linear forms. An easy calculation shows that $\kappa(d,n) \geq \left\lceil \frac{1}{n+1}\binom{n+d}{d} \right\rceil$, and a famous theorem of J. E. Alexander and A. Hirschowitz [J. Algebraic Geom. 4 (1995), no. 2, 201–222; MR1311347 (96f:14065)] says that equality holds unless $d = 2$ or the pair $(d,n)$ is one of the following: $(4,2)$, $(4,3)$, $(3,4)$, $(4,4)$. Geometrically speaking, the above problem amounts to computing the dimensions of the higher secant varieties $\sigma_s(V_{n,d})$, where $V_{n,d} = \nu_d(\mathbb{P}^n)$ is the $d$th Veronese variety, $\nu_d\colon \mathbb{P}^n \hookrightarrow \mathbb{P}^{\binom{n+d}{d}-1}$ is the map defined by the complete linear system of forms of degree $d$ on $\mathbb{P}^n$, and, for a projective variety $X \subset \mathbb{P}^N$, $\sigma_s(X)$ denotes the $s$th self-join of $X$ or, equivalently, the $(s-1)$st secant variety of $X$ defined as the locus of linear subspaces spanned by $s$ points of $X$. In view of the Terracini lemma, the above problem is also equivalent to computing the dimension of the linear system of hypersurfaces of degree $d$ in $n+1$ variables that are singular at a general collection of $s$ points of $\mathbb{P}^n$.

A similar problem of finding out when the higher secant varieties have the expected dimension has been studied for other important series of projective varieties, such as Segre varieties and Grassmann varieties. To get a generalization of the functional Waring problem, one needs to study higher secant varieties of the tangential and higher-order osculating varieties of the Veronese variety $V_{n,d}$. J. Bronowski seems to have been the first to observe that computing the dimensions of the higher secant varieties of the $k$th osculating variety $O_{k,n,d}$ of $V_{n,d}$ allows one to find the minimal number $\kappa = \kappa(k,d,n)$ such that a general form $F$ of degree $d$ in $n+1$ variables can be represented as $F = L_1^{d-k}F_1 + \cdots + L_\kappa^{d-k}F_\kappa$, where $L_1, \ldots, L_\kappa$ are linear forms and $F_1, \ldots, F_\kappa$ are forms of degree $k$ (as expected, for $k = 0$ one arrives at the functional Waring problem). Special cases of the problem of describing the triples $(k,d,n)$ for which $\kappa(k,d,n)$ is larger than expected were considered by A. Inshakov, E. Ballico, A. Geramita, and the authors.

Based on their previous results and using COCOA, in the paper under review the authors make a further contribution to the solution of the above generalized functional Waring problem by proving, among other things, that for $k = 1$ the problem largely reduces to the case $d = 3$ and that for $k = 1$, $n \leq 9$, $\kappa$ is larger than expected if and only if either $d = 2$ or $d = 3$ and $2 \leq n \leq 4$.

(Reviewed by Fyodor L. Zak)