

La preparazione matematica delle matricole nelle cinque sedi del progetto

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Mathematics and Culture in Europe

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with

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and many others

Objective of the test

- **assessment** and **comparison** of actual level of mathematical knowledge of first-year mathematics (and engineering, physics and natural science) students
- stimulate **discussion on high school curricula** in member states
- help **devise policy of teaching** students in their first-year of study
- **facilitate mobility** by giving an insight into guiding students and educators where students desire to switch countries

Differences in and between member states

- different types of schools in countries (e.g. Italy and France)
- number of hours in mathematics in the last three years of high school
- curriculum in mathematics
- university entrance is competitive or not

Differences between member states

Country	Minimum entrance age	Number of school years	Competitive entrance
Italy	19	13	no
Germany	19	13	no
England	18	13	yes
Cyprus	18	12	yes
France	18	12	no

SYLLABUS DI MATEMATICA

Conoscenze e capacità per l'accesso all'Università

*Suggerimenti dell'Unione Matematica Italiana
per la preparazione all'accesso alle Facoltà scientifiche*

1999

Design of the questionnaire:

Test based on the background and skills of mathematics which one should **expect** from a beginner at university in mathematics, science or engineering with questions from **seven areas**:

- Logarithms and exponential (questions 2 and 9);
- Equations and inequalities (questions 4 and 11) ;
- Logic (questions 1 and 8);
- Differential calculus (questions 6 and 13);
- Integral calculus (questions 7 and 14);
- Geometry (questions 5 and 12);
- Trigonometry (questions 3 and 10).

Logarithms and exponential

2. The solution of the equation $\log_2(\log_3 x) = 3$ is

(a) $x = 3$

(b) $x = 3^4$

(c) $x = 3^6$

(d) $x = 3^8$

(e) none of the above answers is correct

9. The number $\sqrt{0.9}$ is equal to

(a) 0.3

(b) 0.81

(c) a number between 0.81 and 0.9

(d) a number between 0.9 and 1

(e) none of the above answers is correct

Equations and inequalities

4. The inequality $\frac{x^2 - 1}{x} > 0$ holds

(a) for each $x \neq 0$

(b) only for $x > 1$

(c) only for $x < -1$

(d) only for $x < -1$ and for $x > 1$

(e) none of the above answers is correct

11. The following fractions $\frac{3}{7} + \frac{1}{8}$ and $\frac{1}{\sqrt{3}-1} + \frac{1}{\sqrt{3}+1}$ expressed in the form $m + n\sqrt{3}$ are equal to

(a) $4/5$ and $\frac{1}{3}\sqrt{3}$ respectively

(b) $31/56$ and $\frac{1}{3}\sqrt{3}$ respectively

(c) $4/5$ and $\sqrt{3}$ respectively

(d) $31/56$ and $\sqrt{3}$ respectively

(e) none of the above answers is correct

Logic

1. The product of seven integers is negative. This implies that
 - (a) all of the numbers are negative
 - (b) one is negative and the others are positive
 - (c) three are negative and the others are positive
 - (d) five are negative and the others are positive
 - (e) none of the above answers is correct

8. The phrase “it is not true that all students are diligent” is equivalent to the phrase

- (a) all students are not diligent
- (b) at least one student is not diligent
- (c) no student is diligent
- (d) at least one student is diligent
- (e) none of the above answers is correct

Differential calculus

6. The coordinates and nature of the turning points on $y = 36x - 3x^2 - 2x^3$ are

- (a) $(-2, -68)$ is a minimum and $(3, 27)$ is a maximum
- (b) $(2, 44)$ is a minimum and $(-3, -81)$ is a maximum
- (c) $(2, 44)$ is a maximum and $(-3, -81)$ is a minimum
- (d) $(-2, -68)$ is a maximum and $(3, 27)$ is a minimum
- (e) none of the above answers is correct

13. The derivative of $(1 - x^2) \ln(1 - x^2)$ with respect to x is

(a) $-2x + 2x \ln(1 - x^2)$

(b) $2x - 2x \ln(1 - x^2)$

(c) $-2x + 2x^2 \ln(1 - x^2)$

(d) $1 - 2x \ln(1 - x^2)$

(e) none of the above answers is correct

Integral calculus

7. Using integration by parts, the integral $\int_0^{\pi} x \sin x \, dx$ is

(a) $\frac{1}{2} \sin(\pi^2)$

(b) -2

(c) $-\pi$

(d) 0

(e) none of the above answers is correct

14. The integral $\int_{-2/3}^{-1/3} (3x + 2)^n dx$ ($n > 1$) is

(a) $\frac{1}{n+1}$

(b) $\frac{3}{n}$

(c) $\frac{1}{3(n-1)}$

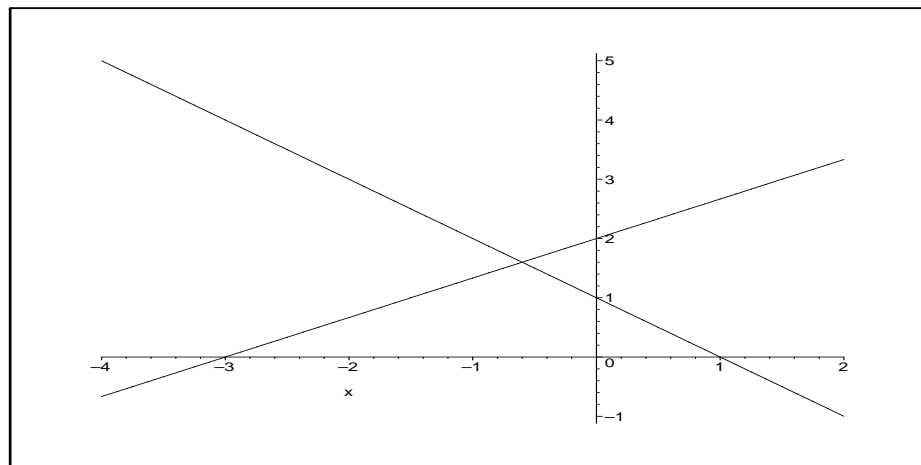
(d) $\frac{1}{3(n+1)}$

(e) none of the above answers is correct

Geometry

5. A triangle ABC has the angles in B and C of 30° and two sides of 40 cm. Relative to the side BC the height is equal to
- (a) $10\sqrt{3}$ cm
 - (b) 20 cm
 - (c) $20\sqrt{3}/3$ cm
 - (d) 80 cm
 - (e) none of the above answers is correct

12. The two lines in the graph



meet at

- (a) $x = -1$ and $y = 2$ (c) $x = -3/5$ and $y = 8/5$
(b) $x = -2/3$ and $y = 5/3$ (d) $x = -11/20$ and $y = 31/20$
(e) none of the above answers is correct

Trigonometry

3. The equation $\sin(2x) = 2 \sin x$ holds

(a) for each x

(b) only for $x = 2k\pi$ with k an arbitrary integer

(c) only for $x = k\pi$ with k an arbitrary integer

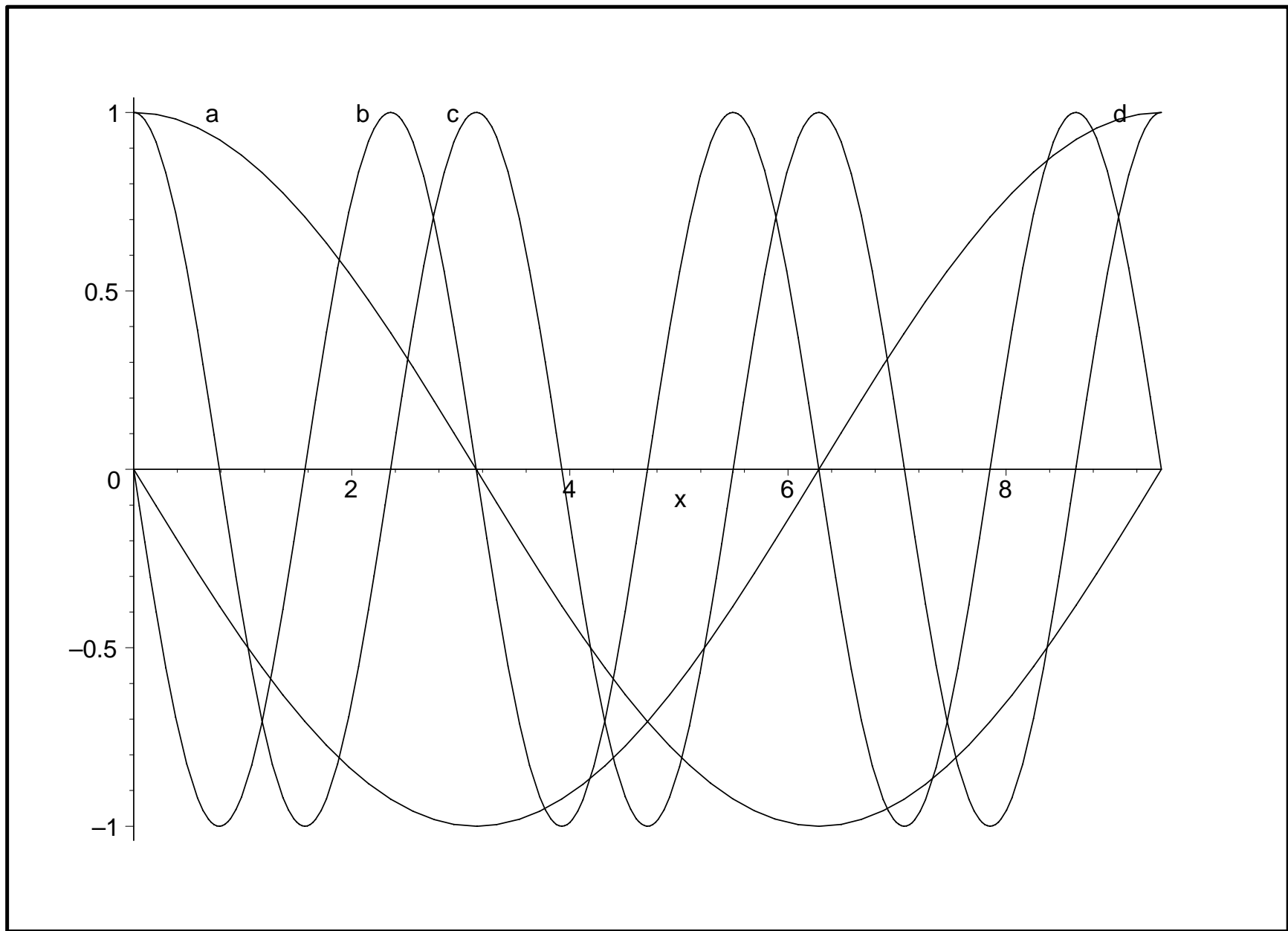
(d) for no value of x

(e) none of the above answers is correct

10. Which of the following graphs

(a) (b) (c) (d) (e) none of them

are that of the function $\sin(2x + \pi/2)$?



- to test technical and computational skills
- to reduce the possibility of students guessing by a “none of the above answer is correct” option
- to trap standard errors
- to require no calculator
- to be easy to mark
- to test the seven different themes in the first seven questions.

Participants

University	Number of students
Cyprus	196
Durham	392
Paris 7	46
Bordeaux	60
Bochum	316
Bayreuth	221
Freiburg	350
Bologna	1648
Catania	275
Total	3441

Discipline	Correct answers (in %)
Physics	57.2
Computer Sciences (Cesena)	54.3
Mathematics	48.8
Industrial Chemistry (Bologna)	48.6
Biotechnology	43.9
Statistical Sciences (Bologna)	40.9
Chemistry and Chemistry of Materials	40.4
Astronomy	39.8
Informatics (M–Z)	38.6
Chemistry and Pharmaceutical Techniques	36.7
Pharmacy	33.0
Chem. Techn. Environm. Waste Management	32.5
Internet Sciences	31.3
Economics	29.4
Natural Sciences	26.6
Biological Sciences	25.8
Agriculture (Imola)	22.4
Industrial Chemistry (Faenza)	22.1
Agriculture	10.4

Discipline	Correct answers (in %)				
	Italy	Germany	UK	Cyprus	France
Physics	57.2 47.0 (Catania)			42.0	
Science		32.4 49.0 (Bayreuth) 47.5 (Freiburg)			55.1
Computer Sciences	54.3 (Cesena)			49.9	
Mathematics	48.8 42.1 (Catania)	45.2 47.9 (Freiburg)	67.0	55.6	
Industrial Chemistry	48.6 (Bologna)				
Biotechnology	43.9				
Statistical Sciences	40.9 (Bologna)				
Chemistry/Chem. of Materials	40.4			46.1	
Astronomy	39.8				
Informatics	38.6				
Chem. Pharmaceutical Techniques	36.7				
Special Architecture	36.6 (Catania)				
Pharmacy	33.0				
Chem. Techn. Envir. Waste Management	32.5 (Rimini)				
Internet Sciences	31.3				
Economics	29.4				
Natural Sciences	26.6		61.3		
Biological Sciences	25.8				
Agriculture	22.4 (Imola)				
Industrial Chemistry	22.1 (Faenza)				
Agriculture	10.4				
Engineering	45.5 (Catania)	37.5	56.3	46.9	
Other		31.9 51.7 (Freiburg)	54.6		
Classics (History of Science and Logic)					21.4 (Bordeaux)

Students of Mathematics

University	No.	Correct answers per question (in %)						
		Qu 1	Qu 2	Qu 3	Qu 4	Qu 5	Qu 6	Qu 7
Bologna	74	51	57	18	55	64	36	18
Bochum	54	63	37	31	7	48	63	24
Freiburg	107	79	27	24	16	44	52	20
Durham	105	79	65	39	22	76	77	56
Cyprus	38	39	81	47	39	78	28	31
Paris 7	46	63	93	61	76	22	72	61
All	424	67	55	34	32	57	57	35

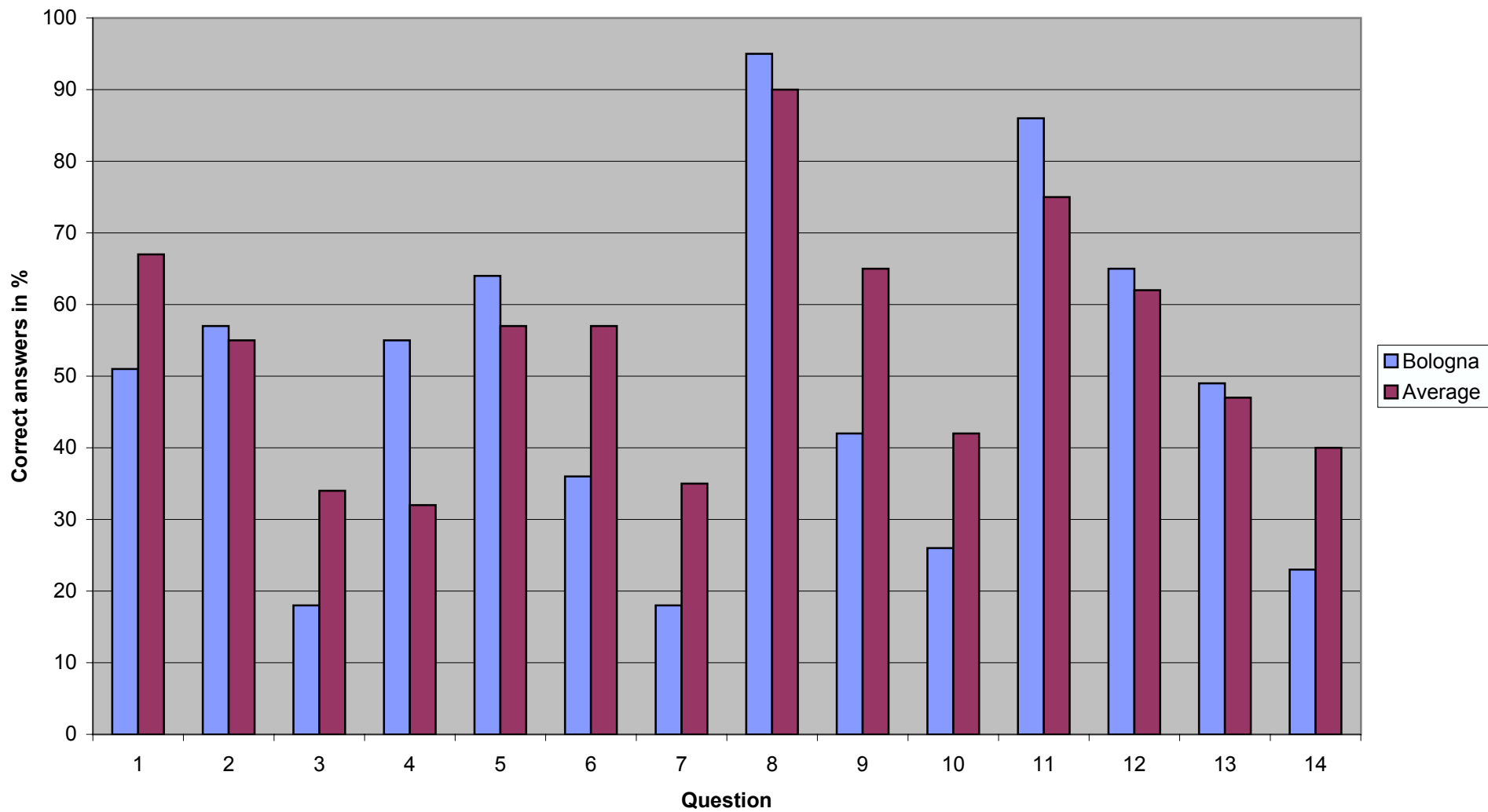
weakest strongest

Students of Mathematics

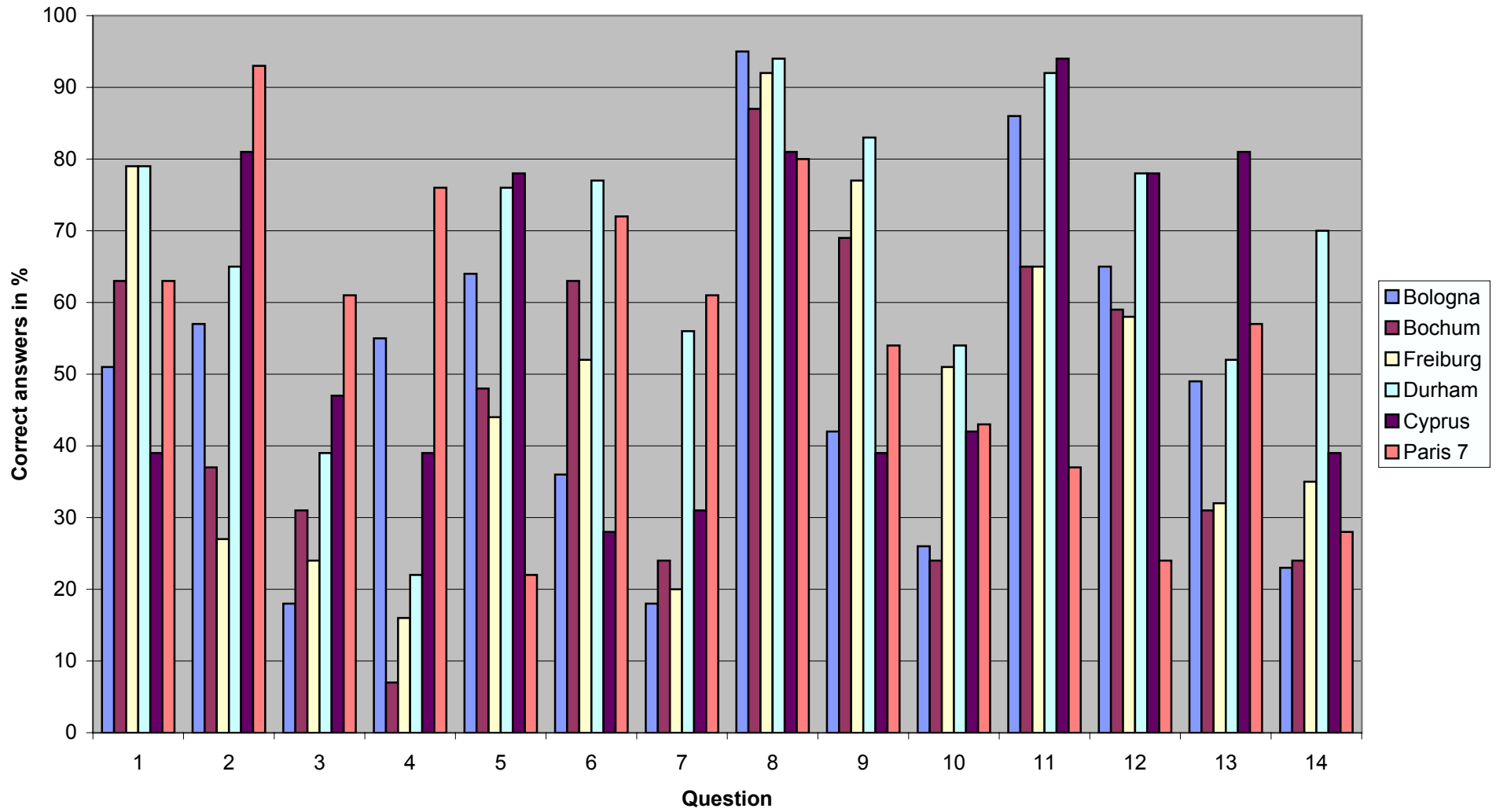
University	No.	Correct answers per question (in %)						
		Qu 8	Qu 9	Qu 10	Qu 11	Qu 12	Qu 13	Qu 14
Bologna	74	95	42	26	86	65	49	23
Bochum	54	87	69	24	65	59	31	24
Freiburg	107	92	77	51	65	58	32	35
Durham	105	94	83	54	92	78	52	70
Cyprus	38	81	39	42	94	78	81	39
Paris 7	46	80	54	43	37	24	57	28
All	424	90	65	42	75	62	47	40

weakest strongest

Math Students



Math Students



Results – conclusions

- no significant gender imbalance
- striking difference between students of universities with and without entry examinations
- too many students of disciplines with a service course in mathematics not sufficiently prepared (Bologna, Bochum)
- weaknesses in trigonometry, differentiation (in some countries question 6, in other countries question 13) and integration

- best results where numeracy was tested (question 11)
- students' mobility between the test countries is not constrained by their mathematical preparation
- similarity to the **PISA** (OECD **P**rogramme for **I**nternational **S**tudent **A**ssessment) ranking of mathematical literacy of fifteen-year-old students from 2000

560
555
550
545
540
535
530
525
520
515
510
505
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460
455
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340
335
330
325

