(1) $\dot{x} + t x \overset{\bullet}{=} e^{-t^2/2} \cdot t \qquad A(t) = \int_0^t s\,ds = \frac{t^2}{2}$

$$e^{t^2/2}\dot{x} + e^{t^2/2} \cdot t \cdot x \overset{\bullet}{=} t$$

$$\underbrace{}_{\parallel}$$

$$\frac{d}{dt}\left[e^{t^2/2} \cdot x\right] \Rightarrow e^{t^2/2} \cdot x(t) = \frac{t^2}{2} + c$$

$$\boxed{x(t) = e^{-t^2/2}\left(\frac{t^2}{2} + c\right)}$$

(2) $\dot{x} + \sin(t)\,x \overset{\bullet}{=} e^{\cos(t)} \cdot \cos(t) \cdot \sin(t) \qquad A(t) = \int_{\pi/2}^t \sin(s)\,ds = -\cos(t)$

$$e^{-\cos t}\dot{x} + e^{-\cos t} \cdot \sin t \cdot x \overset{\bullet}{=} \cos(t) \cdot \sin(t)$$

$$\frac{d}{dt}\left[e^{-\cos t} \cdot x\right] \Rightarrow e^{-\cos t} \cdot x(t) - e^{-\cos(\pi/2)} \cdot x(\pi/2) \overset{\bullet}{=} \int_{\pi/2}^t \cos(s)\sin(s)\,ds$$

$$e^{-\cos t} \cdot x(t) - e^{-1} \qquad \frac{\sin^2(t)-1}{2} = \left[\frac{\sin^2(t)}{2}\right]_{\pi/2}^{t}$$

$$\Rightarrow \boxed{x(t) = e^{\cos t}\left(e^{-1} + \frac{\sin^2(t)-1}{2}\right)}$$

(3) $\boxed{x(t) = c \cdot e^{-t^2/2}}$

(4) Posto $\dot{x} = y$ ho che $\dot{y} - y \overset{\bullet}{=} t$: $A(t) = \int_0^t (-1)\,ds = -t$

$$e^{-t}\dot{y} - e^{-t}y \overset{\bullet}{=} e^{-t}t \Rightarrow e^{-t}y(t) \overset{\bullet}{=} c + \int_0^t e^{-s}s\,ds = \left[-e^{-s}s\right]_0^t + \int_0^t e^{-s}\,ds + c$$

$$\frac{d}{dt}\left[e^{-t}y\right]$$

$$= c - e^{-t}t - e^{-t} + 1 = 1 - e^{-t}(1+t) + c$$

$$= k - e^{-t}(1+t)$$

$$\Rightarrow y(t) = k e^{+t} - (1+t)$$

$$\Rightarrow \boxed{x(t) = k e^t - \left(t + \frac{t^2}{2}\right) + H} \qquad k, H \in \mathbb{R}$$

(5) $\dot{x} + \frac{1}{1+t^2} \cdot x \overset{\bullet}{=} \frac{f(t)}{1+t^2} \qquad A(t) = \int_0^t \frac{ds}{1+s^2} = \operatorname{arctg}(t)$

$$\frac{d}{dt}\left[e^{\operatorname{arctg}(t)}x(t)\right] = e^{\operatorname{arctg}(t)}\dot{x}(t) + e^{\operatorname{arctg}(t)}\frac{1}{1+t^2}x(t) \overset{\bullet}{=} \frac{e^{\operatorname{arctg}(t)}}{1+t^2}f(t)$$

$$e^{\operatorname{arctg}(t)}x(t) = \int_0^t \frac{e^{\operatorname{arctg}(s)}}{1+s^2} f(s)\,ds + k$$

$$\boxed{x(t) = e^{-\operatorname{arctg}(t)} \cdot \int_0^t \frac{e^{\operatorname{arctg}(s)}}{1+s^2} \cdot f(s)\,ds + k \cdot e^{-\operatorname{arctg}(t)}}$$

(6) Rifaccio i conti di (5), ma integro tra 1 e t:

$$A(t) = \int_1^t \frac{ds}{1+s^2} = \operatorname{arctg}(t) - \operatorname{arctg}(1) = \operatorname{arctg}(t) - \pi/4$$

$$\frac{d}{dt}\left[e^{\operatorname{arctg}(t)-\pi/4} \cdot x(t)\right] \overset{\circ}{=} \frac{e^{\operatorname{arctg}(t)-\pi/4}}{1+t^2} f(t)$$

$$e^{\operatorname{arctg}(t)-\pi/4} x(t) - e^{\operatorname{arctg}(1)-\pi/4} \cdot x(1) \overset{\circ}{=} \int_1^t \frac{e^{\operatorname{arctg}(s)-\pi/4}}{1+s^2} \cdot f(s)\, ds$$

$$e^{\operatorname{arctg}(t)-\pi/4} x(t) - e^{0} \cdot 0$$

$$\Rightarrow \boxed{x(t) = e^{-\operatorname{arctg}(t)+\pi/4} \cdot \int_1^t \frac{e^{\operatorname{arctg}(s)-\pi/4}}{1+s^2} f(s)\, ds = e^{-\operatorname{arctg}(t)} \cdot \int_1^t \frac{e^{\operatorname{arctg}(s)}}{1+s^2} f(s)\, ds}$$

(7) $$\frac{d}{dt}\left[\frac{d}{dt}(e^t x) + e^t x\right] \overset{\circ}{=} 1$$

$$\frac{d}{dt}(e^t x) + e^t x \overset{\circ}{=} t + c$$

Posto $y = e^t x$: $\quad \dot{y} + y \overset{\circ}{=} t + c \qquad A(t) = \int_0^t 1 \cdot ds = t$

$$\frac{d}{dt}\left[e^t y\right] = e^t \dot{y} + e^t y \overset{\circ}{=} e^t (t+c)$$

$$e^t y \overset{\circ}{=} \int_0^t e^s (s+c)\, ds + D = \left[e^s \cdot (s+c)\right]_0^t + \int_0^t e^s\, ds + D$$

$$= e^t (t+c) - c + e^t - 1 + D$$

$$= e^t (t+c+1) + (D-c-1) = e^t (t+H) + K \text{ dove } \begin{cases} H = c+1 \\ K = D-c-1 \end{cases}$$

$$\boxed{y(t) = t + H + K e^{-t} \quad \text{con} \quad H, K \in \mathbb{R}}$$