Equazioni differenziali lineari del secondo ordine a coefficienti costanti

Determinare l'integrale generale delle seguenti equazioni differenziali lineari del secondo ordine a coefficienti costanti, o risolvere il relativo problema di Cauchy.

Equazioni omogenee

(a) $y'' + y' + y = 0\,;$ $\left[\left\{t \mapsto e^{-\frac{t}{2}}\left(c_1 \cos\frac{\sqrt{3}\,t}{2} + c_2 \sin\frac{\sqrt{3}\,t}{2}\right) \mid c_1, c_2 \in \mathbb{R}\right\}\right]$

(b) $y'' = -\sqrt{2}\,y'\,;$ $\left[\left\{t \mapsto c_1 + c_2\, e^{-\sqrt{2}\,t} \mid c_1, c_2 \in \mathbb{R}\right\}\right]$

(c) $y'' - \dfrac{y'}{2} + \dfrac{y}{16} = 0\,;$ $\left[\left\{t \mapsto (c_1 + c_2 t)\, e^{\frac{t}{4}} \mid c_1, c_2 \in \mathbb{R}\right\}\right]$

(d) $\begin{cases} y'' - y' = 2y\,, \\ y(0) = 0\,, \\ y'(0) = 3\,; \end{cases}$ $\left[y(t) = e^{2t} - e^{-t}\right]$

(e) $\begin{cases} y'' = 6y' - 10y\,, \\ y(0) = 1\,, \\ y'(0) = 0\,; \end{cases}$ $\left[y(t) = e^{3t}(\cos t - 3\sin t)\right]$

(f) $\begin{cases} 25y = 10y' - y''\,, \\ y(0) = 0\,, \\ y'(0) = 1\,. \end{cases}$ $\left[y(t) = te^{5t}\right]$

Equazioni complete

(a) $y'' - y = \cos t\,;$ $\left[\left\{t \mapsto c_1 e^{-t} + c_2 e^{t} - \frac{1}{2}\cos t \mid c_1, c_2 \in \mathbb{R}\right\}\right]$

(b) $y'' + y = te^{-t}\,;$ $\left[\left\{t \mapsto c_1 \cos t + c_2 \sin t + \frac{1}{2}(t+1)e^{-t} \mid c_1, c_2 \in \mathbb{R}\right\}\right]$

(c) $y'' - 2y' + y = t^2 - 6\,;$ $\left[\left\{t \mapsto e^{t}(c_1 + c_2 t) + t^2 + 4t \mid c_1, c_2 \in \mathbb{R}\right\}\right]$

(d) $y'' - 2y' - 3y = 8e^{3t}\,;$ $\left[\left\{t \mapsto c_1 e^{-t} + c_2 e^{3t} + 2te^{3t} \mid c_1, c_2 \in \mathbb{R}\right\}\right]$

$$(e)\quad \begin{cases} y'' + 4y = t\,, \\ y(0) = 1\,, \\ y'(0) = 0\,; \end{cases} \qquad \left[y(t) = \cos(2t) - \frac{1}{8}\sin(2t) + \frac{t}{4}\right]$$

$$(f)\quad \begin{cases} y'' = -9y + (39t - 12)\,e^{-2t}\,, \\ y(1) = 3e^{-2}\,, \\ y'(1) = -3e^{-2}\,; \end{cases} \qquad \left[y(t) = 3te^{-2t}\right]$$

$$(g)\quad \begin{cases} y'' - 2y' - 5 = 0\,, \\ y(2) = y'(2) = -\dfrac{5}{2}\,; \end{cases} \qquad \left[y(t) = \frac{5}{2}(1-t)\right]$$

$$(h)\quad \begin{cases} y'' = y' + 2y + 2\sin t\,, \\ y(0) = \dfrac{2}{5}\,, \\ y'(0) = -\dfrac{1}{5}\,. \end{cases} \qquad \left[y(t) = \frac{1}{5}\left(e^{2t} + \cos t - 3\sin t\right)\right]$$