## Prodotti di convoluzione

Determinare $u * v$, posti

1. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = \chi_{[-1/2,\,1/2]}(t), \quad v(t) = H(t);$
2. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = e^t \chi_{(-\infty,0]}(t), \quad v(t) = e^{-t} \chi_{[0,\infty)}(t);$
3. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = \chi_{[1,2]}(t), \quad v(t) = (t+2)\chi_{[0,\infty)}(t);$
4. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = e^t \chi_{(-\infty,0]}(t), \quad v(t) = \sin t;$
5. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = t\,\chi_{[0,1]}(t), \quad v(t) = \cos t;$
6. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = \frac{1}{t} H(t-1), \quad v(t) = \frac{1}{t} H(t+1);$
7. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = \frac{t}{t^2+1}, \quad v(t) = \chi_{[-1,1]}(t);$
8. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = v(t) = \frac{t}{t^2+1}\,;$
9. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = (1 - |t|)^+, \quad v(t) = \chi_{[-1/2,\,1/2]}(t);$
10. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = v(t) = (1 - |t|)^+;$
11. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = \alpha\,\chi_{[-\alpha,0]}(t), \quad v(t) = t\,\chi_{[0,\alpha]}(t),$ $\alpha > 0;$
12. $u, v : \mathbb{R} \to \mathbb{R}, \qquad u(t) = \chi_{[-\alpha,\alpha]}(t), \quad v(t) = (\alpha + t)\,\chi_{[-\alpha,0]}(t),$ $\alpha > 0.$

### Risultati

1. $$(u * v)(x) = \begin{cases} 0, & \text{se } x \le -\frac{1}{2}\,, \\ x + \frac{1}{2}\,, & \text{se } -\frac{1}{2} < x < \frac{1}{2}\,, \\ 1, & \text{se } x \ge \frac{1}{2}\,; \end{cases}$$
2. $(u * v)(x) = \frac{1}{2}\, e^{-|x|}, \quad x \in \mathbb{R};$
3. $$(u * v)(x) = \begin{cases} 0, & \text{se } x \le 1, \\ \frac{1}{2}(x^2 + 2x - 3), & \text{se } 1 < x \le 2, \\ x + \frac{1}{2}\,, & \text{se } x > 2; \end{cases}$$

4. $(u*v)(x) = \frac{1}{2}\,(\sin x + \cos x), \quad x \in \mathbb{R};$

5. $(u*v)(x) = \cos(x-1) - \sin(x-1) - \cos x, \quad x \in \mathbb{R};$

6. $(u*v)(x) = \begin{cases} 0, & \text{se } x \leq 0, \\ \frac{1}{x}\ln|x^2-1|, & \text{se } x > 0,\, x \neq 1; \end{cases}$

7. $(u*v)(x) = \frac{1}{2}\ln\left|\frac{x^2+2x+2}{x^2-2x+2}\right|, \quad x \in \mathbb{R};$

8. $(u*v)(x) = \frac{-2\pi}{x^2+4}\,, \quad x \in \mathbb{R};$

9. $(u*v)(x) = \begin{cases} 0, & \text{se } x \leq -\frac{3}{2} \cup x > \frac{3}{2}\,, \\ \frac{1}{8}(2x+3)^2, & \text{se } -\frac{3}{2} < x \leq -\frac{1}{2}\,, \\ -x^2+\frac{3}{4}\,, & \text{se } -\frac{1}{2} < x \leq \frac{1}{2}\,, \\ \frac{1}{8}(2x-3)^2, & \text{se } \frac{1}{2} < x \leq \frac{3}{2}\,; \end{cases}$

10. $(u*v)(x) = \begin{cases} 0, & \text{se } x \leq -1 \cup x > 2, \\ \frac{1}{6}(x+1)^3, & \text{se } -1 < x \leq 0, \\ -\frac{1}{3}x^3+\frac{1}{2}x+\frac{1}{6}\,, & \text{se } 0 < x \leq 1, \\ \frac{1}{6}x^3-\frac{1}{2}x^2+\frac{2}{3}, & \text{se } 1 < x \leq 2; \end{cases}$

11. $(u*v)(x) = \begin{cases} 0, & \text{se } x \leq -\alpha \cup x \geq \alpha, \\ \frac{\alpha}{2}(x+\alpha)^2, & \text{se } -\alpha < x < 0, \\ \frac{\alpha}{2}(\alpha^2-x^2), & \text{se } 0 < x < \alpha; \end{cases}$

12. $(u*v)(x) = \begin{cases} 0, & \text{se } x \leq -2\alpha \cup x > \alpha, \\ \frac{1}{2}(x+2\alpha)^2, & \text{se } -2\alpha < x \leq -\alpha, \\ \frac{1}{2}\,\alpha^2, & \text{se } -\alpha < x \leq 0, \\ \frac{1}{2}(\alpha^2-x^2), & \text{se } 0 < x \leq \alpha. \end{cases}$