Applicazioni delle trasformate di Laplace

Risolvere i seguenti problemi di Cauchy.

1. $\begin{cases} x''(t) - 4\,x(t) = e^{2t}\sin t\,, & t \geq 0\,, \\ x(0) = 0\,, \\ x'(0) = 0\,; \end{cases}$

2. $\begin{cases} x''(t) + 6\,x'(t) + 9\,x(t) = 9\,e^{3t}\,, & t \geq 0\,, \\ x(0) = 0\,, \\ x'(0) = 0\,; \end{cases}$

3. $\begin{cases} x''(t) - 3\,x'(t) + 2\,x(t) = e^{5t}\,, & t \geq 0\,, \\ x(0) = 1\,, \\ x'(0) = 2\,; \end{cases}$

4. $\begin{cases} x''(t) + 8\,x'(t) + 52\,x(t) = t\,, & t \geq 0\,, \\ x(0) = 2\,, \\ x'(0) = 0\,. \end{cases}$

Classificazione dei punti singolari isolati

Determinare e classificare le singolarita' delle seguenti funzioni.

1. $f(z) = \dfrac{z^2+1}{z(z-1)(z-i)^2}\,;$

2. $f(z) = z\,\sin(1/z)\,;$

3. $f(z) = \dfrac{1}{\sin z}\,;$

4. $f(z) = \dfrac{1-e^{-z}}{z(z^2+3)}\,;$

5. $f(z) = \dfrac{1}{z}\,e^{1/z}\,;$

6. $f(z) = \dfrac{z(z-i\pi)}{\sinh z}\,;$

7. $f(z) = \dfrac{\pi \cos(\pi z)}{z^2 \sin(\pi z)}$;

8. $f(z) = \dfrac{1}{\cosh z}$.

Integrali con il teorema dei residui

Calcolare il valore dei seguenti integrali.

1. $\displaystyle\int_0^\infty \frac{x^2}{x^4+x^2+1}\,dx$; $\left[\dfrac{\pi\sqrt{3}}{6}\right]$

2. $\displaystyle\int_{-\infty}^\infty \frac{x}{(x^2+1)(x^2+2x+3)}\,dx$; $\left[\dfrac{\pi(1-\sqrt{2})}{4}\right]$

3. $\displaystyle\int_{-\infty}^\infty \frac{x^2}{1+x^4}\,dx$; $\left[\dfrac{\pi}{\sqrt{2}}\right]$

4. $\displaystyle\int_0^\infty \frac{x^2}{x^4+3x^2+2}\,dx$; $\left[\dfrac{\pi}{\sqrt{2}} - \dfrac{\pi}{2}\right]$

5. $\displaystyle\int_0^\infty \frac{x^2}{(x^2+4)^2(x^2+9)}\,dx$; $\left[\dfrac{\pi}{200}\right]$

6. $\displaystyle\int_0^\infty \frac{x^2}{(1+x^2)^2}\,dx$; $\left[\dfrac{\pi}{4}\right]$

7. $\displaystyle\int_{-\infty}^\infty \frac{1}{x^4+x^2+1}\,dx$; $\left[\dfrac{\pi}{\sqrt{3}}\right]$

8. $\displaystyle\int_{-\infty}^\infty \frac{x+4}{(x^2+3)^2(x^2+x+2)}\,dx$;

9. $\displaystyle\int_{-\infty}^\infty \frac{x^2+\alpha\, i}{(x+i)^2(x^2+\beta^2)^2}\,dx$, $\quad \alpha,\beta>0$; $\left[\dfrac{\pi}{2}\dfrac{\beta^3-\beta^2-\alpha(3\beta+1)i}{\beta^3(\beta+1)^3}\right]$

10. $\displaystyle\int_{-\infty}^\infty \frac{e^{-\alpha\, i\, x}}{(x^2+\alpha)(x^2+x+1)}\,dx$, $\quad \alpha>0$;

11. $\displaystyle\int_{-\infty}^\infty \frac{e^{\,i\,x}}{x^4+1}\,dx$; $\left[\pi\, e^{-\frac{\sqrt{2}}{2}} \cos\dfrac{2\sqrt{2}-\pi}{4}\right]$

12. $\displaystyle\int_{-\infty}^{\infty} \frac{x^2}{x^4+1}\, e^{-2\,i\,x}\, dx\,;$

13. $\displaystyle\int_{-\infty}^{\infty} \frac{e^{-\,i\,x}}{x^4+x^2+2}\, dx\,;$

14. $\displaystyle\int_{-\infty}^{\infty} \frac{x}{x^2+x+1}\, e^{\,i\,x}\, dx\,;$

15. $\displaystyle\int_{-\infty}^{\infty} \frac{e^{-2\,i\,x}}{(x^2+9)(x^2-x+3)}\, dx\,;$

16. $\displaystyle\int_{-\infty}^{\infty} \frac{e^{-4\,i\,x}}{x^2+x+\frac{17}{4}}\, dx\,;$ $\left[\dfrac{\pi}{2}\, e^{2i-8}\right]$

17. $\displaystyle\int_{-\infty}^{\infty} \frac{\cos x}{1+x^2}\, dx\,.$ $\left[\dfrac{\pi}{e}\right]$