Trasformate di Laplace

Calcolare la trasformata di Laplace delle seguenti funzioni e determinarne l'ascissa di convergenza.

1. $f : \mathbb{R} \to \mathbb{R}, \qquad f(t) = \begin{cases} 0\,, & \text{se } t \in (-\infty, 2)\,, \\ (t-2)\sinh(4t)\,, & \text{se } t \in [2, +\infty)\,; \end{cases}$

2. $f : \mathbb{R} \to \mathbb{R}, \qquad f(t) = \begin{cases} 0\,, & \text{se } t \in (-\infty, 0)\,, \\ \sin t\,, & \text{se } t \in [0, \pi]\,, \\ (t-\pi)^4\,, & \text{se } t \in (\pi, +\infty)\,; \end{cases}$

3. $f : \mathbb{R} \to \mathbb{R}, \qquad f(t) = \begin{cases} 0\,, & \text{se } t \in (-\infty, 0)\,, \\ t\,, & \text{se } t \in [0, 1]\,, \\ t^2, & \text{se } t \in (1, 2]\,, \\ 4\,, & \text{se } t \in (2, +\infty)\,; \end{cases}$

4. $f : \mathbb{R} \to \mathbb{R}, \qquad f(t) = t(t-1)H(t-1)e^{2t};$

5. $f : \mathbb{R} \to \mathbb{R}, \qquad f(t) = t(t-1)e^{2t};$

6. $f : \mathbb{R} \to \mathbb{R}, \qquad f(t) = tH(t-\alpha)e^{\beta t}, \qquad \alpha, \beta > 0\,;$

7. $f : \mathbb{R} \to \mathbb{R}, \qquad f(t) = \left(\frac{1}{2}\sin(4t) - 4\cosh(2t)\right) e^{-3t};$

8. $f : \mathbb{R} \to \mathbb{R}, \qquad f(t) = \begin{cases} 0\,, & \text{se } t \in (-\infty, 0)\,, \\ (t^2 + \alpha t)e^{\alpha t}, & \text{se } t \in [0, +\infty)\,, \end{cases} \qquad \alpha > 0\,;$

9. $f : \mathbb{R} \to \mathbb{R}, \qquad f(t) = \begin{cases} 0\,, & \text{se } t \in (-\infty, 0)\,, \\ (2t+5)e^{3t}, & \text{se } t \in [0, +\infty)\,; \end{cases}$

10. $f : \mathbb{R} \to \mathbb{R}, \qquad f(t) = \begin{cases} 0\,, & \text{se } t \le a\,, \\ e^t, & \text{se } a < t \le b\,, \\ b\,, & \text{se } t > b\,, \end{cases} \qquad a, b > 0\,;$

11. $f : \mathbb{R} \to \mathbb{R}, \qquad f(t) = \begin{cases} 0\,, & \text{se } t \in (-\infty, \pi)\,, \\ |\sin t|\,, & \text{se } t \in [\pi, +\infty)\,; \end{cases}$

12. $f:[0,+\infty)\to\mathbb{R}$, $\quad f(t)=\begin{cases} t\,, & \text{se } t\in[0,\alpha)\,,\\ (t-\alpha)^2\,, & \text{se } t\in(\alpha,2\alpha)\,,\end{cases}$

$f(t+2\alpha k)=f(t)$ per ogni $k\in\mathbb{N}$.

Risultati

1. $\dfrac{e^{8-2s}}{2(s-4)^2}-\dfrac{e^{-8-2s}}{2(s+4)^2}\,,\qquad \text{Re}(s)>4\,;$

2. $\dfrac{1+e^{-\pi s}}{1+s^2}+\dfrac{24\,e^{-\pi s}}{s^5}\,,\qquad \text{Re}(s)>0\,;$

3. $\dfrac{4\,e^{-2s}-e^{-s}}{s}+\dfrac{1-e^{-s}}{s^2}+\dfrac{e^{-s}(s^2+2s+2)-2e^{-2s}(1+2s+2s^2)}{s^3}\,,$
$\text{Re}(s)>0\,;$

4. $\dfrac{2\,e^{2-s}}{(s-2)^3}+\dfrac{e^{2-s}}{(s-2)^2}\,,\qquad \text{Re}(s)>2\,;$

5. $\dfrac{2}{(s-2)^3}-\dfrac{1}{(s-2)^2}\,,\qquad \text{Re}(s)>2\,;$

6. $\dfrac{\alpha\,e^{\alpha\beta-\alpha s}}{s-\beta}+\dfrac{e^{\alpha\beta-\alpha s}}{(s-\beta)^2}\,,\qquad \text{Re}(s)>\beta\,;$

7. $\dfrac{2}{(s+3)^2+16}-\dfrac{4(s+3)}{(s+3)^2-4}\,,\qquad \text{Re}(s)>-1\,;$

8. $\dfrac{2}{(s-\alpha)^3}+\dfrac{\alpha}{(s-\alpha)^2}\,,\qquad \text{Re}(s)>\alpha\,;$

9. $\dfrac{2}{(s-3)^2}+\dfrac{5}{s-3}\,,\qquad \text{Re}(s)>3\,;$

10. $\begin{cases} \frac{b\,e^{-bs}}{s}+\frac{e^{a-as}-e^{b-bs}}{s-1}\,, & \text{se } s\neq 1\,,\\ be^{-b}+b-a\,, & \text{se } s=1\,,\end{cases}\qquad \text{Re}(s)>0\,;$

11. $\dfrac{e^{-\pi s}(1+e^{-\pi s})}{(s^2+1)(1-e^{-\pi s})}\,,\qquad \text{Re}(s)>0\,;$

12. $$\frac{1}{1-e^{-2\alpha s}}\Big(-\frac{\alpha}{s}(e^{-\alpha s}+\alpha\, e^{-2\alpha s})+\frac{1}{s^2}(1-e^{-\alpha s}-2\alpha\, e^{-2\alpha s}) + \frac{2}{s^3}(e^{-\alpha s}-e^{-2\alpha s})\Big), \qquad \mathrm{Re}(s)>0\,.$$

Antitrasformate di Laplace

Calcolare l'antitrasformata di Laplace delle seguenti funzioni.

1. $K(s) = \dfrac{1}{s^2-2}\,;$

2. $K(s) = \dfrac{s-1}{s^2-s-6}\,e^{-s};$

3. $K(s) = \dfrac{s}{(s+1)^3}\,e^{-4s};$

4. $K(s) = \dfrac{s+3}{s^2+4}\,;$

5. $K(s) = \dfrac{s^2+1}{s(s^2+2)}\,e^{-\frac{s}{3}};$

6. $K(s) = \dfrac{1}{s^2(3s+2)^2}\,;$

7. $K(s) = \dfrac{e^{-4s}}{s^3+3s^2+2s}\,;$

8. $K(s) = \dfrac{e^{-\alpha s}}{s^2-(\alpha+2)s+2\alpha}\,, \qquad \alpha>0\,;$

9. $K(s) = \dfrac{e^{-bs}}{s(s^2+4as+4a^2+b^2)}\,, \qquad a,b>0\,;$

10. $K(s) = \dfrac{e^{-\alpha s}}{s(s^2+\alpha^2)}\,, \qquad \alpha>0\,;$

11. $K(s) = \dfrac{e^{-\beta s}}{s(s+\alpha)^2}\,, \qquad \alpha,\beta>0\,;$

12. $\quad K(s) = \dfrac{e^{-7s}}{(s-7)(s-6)}\,;$

13. $\quad K(s) = \dfrac{3s^2+3s+1}{s^2(s^2+s+3)}\,.$

Risultati

1. $\dfrac{H(t)}{\sqrt{2}}\sinh(\sqrt{2}t)\,;$
2. $H(t-1)\Big(\dfrac{2}{5}\,e^{3(t-1)}+\dfrac{3}{5}\,e^{-2(t-1)}\Big);$
3. $H(t-4)\Big((t-4)e^{4-t}-\dfrac{1}{2}(t-4)^2e^{4-t}\Big);$
4. $H(t)\Big(\cos(2t)+\dfrac{3}{2}\sin(2t)\Big);$
5. $H\Big(t-\dfrac{1}{3}\Big)\Big(\dfrac{1}{2}+\dfrac{1}{2}\cos\Big(\sqrt{2}t-\dfrac{\sqrt{2}}{3}\Big)\Big);$
6. ...
7. ...
8. $\dfrac{H(t-\alpha)}{\alpha-2}\big(e^{\alpha t-\alpha^2}-e^{2t-2\alpha}\big);$
9. $\dfrac{H(t-b)}{4a^2+b^2}\Big(1-e^{2ab-2at}\Big(\dfrac{2a}{b}\sin(bt-b^2))+\cos(bt-b^2)\Big)\Big);$
10. ...
11. $\dfrac{H(t-\beta)}{\alpha}\Big(-e^{\alpha\beta-\alpha t}\Big((t-\beta)+\dfrac{1}{\alpha}\Big)+\dfrac{1}{\alpha}\Big);$
12. ...
13. $H(t)\Big(\dfrac{t}{3}+\dfrac{8}{9}-\dfrac{8}{9}\,e^{-\frac{t}{2}}\cos\Big(\dfrac{\sqrt{11}}{2}\,t\Big)+\dfrac{40}{9\sqrt{11}}\,e^{-\frac{t}{2}}\sin\Big(\dfrac{\sqrt{11}}{2}\,t\Big)\Big).$