

Integrali generalizzati

Stabilire se i seguenti integrali generalizzati sono convergenti:

1. $\int_0^{+\infty} \frac{\ln(1+x)}{x^2} dx;$ [non converge]
2. $\int_0^1 \frac{x^x - 1}{x} dx;$ [converge]
3. $\int_0^1 \frac{\ln(1+x^2)}{\sin x - \sinh x} dx;$ [non converge]
4. $\int_0^{\frac{\pi}{4}} \frac{\cosh^2 x - 1}{\operatorname{arcsinh} x - x} dx;$ [converge]
5. $\int_2^e \frac{\ln \ln x}{(e-x)^{\frac{3}{2}}} dx;$ [converge]
6. $\int_0^{+\infty} \left(\arctan x - \arctan \frac{x^2}{x+1} \right) dx;$ [converge]
7. $\int_1^{+\infty} \left(\left(1 + \frac{1}{x} \right)^x - e \right) dx;$ [non converge]
8. $\int_0^1 \frac{\sin x}{x^\alpha \sqrt{1-x}} dx, \quad \alpha > 0;$ [converge $\Leftrightarrow 0 < \alpha < 2$]
9. $\int_0^1 \frac{e^x \tan(x^3)}{\sqrt[3]{1+x^\alpha} - 1} dx, \quad \alpha > 0;$ [converge $\Leftrightarrow 0 < \alpha < 4$]
10. $\int_0^{+\infty} \frac{\arctan x}{x^\alpha} dx, \quad \alpha > 0;$ [converge $\Leftrightarrow 1 < \alpha < 2$]
11. $\int_0^{+\infty} \frac{x^\alpha (x + \sin x)}{x - \sin x} dx, \quad \alpha > 0;$ [$\nexists \alpha$ per cui converga]
12. $\int_0^{+\infty} \frac{x^{\alpha x} |\sinh x - \cosh^2 x|}{x^{4\alpha}} dx, \quad \alpha \in \mathbb{R};$ [converge $\Leftrightarrow \alpha < 0$]
13. $\int_0^1 x \ln \frac{1+x^\alpha}{1-x} dx, \quad \alpha > 0;$ [converge $\forall \alpha > 0$]
14. $\int_0^{+\infty} \frac{\sin^2 x + 1 - \cos x}{x^\alpha} dx, \quad \alpha > 0;$ [converge $\Leftrightarrow 1 < \alpha < 3$]

15. $\int_0^{+\infty} \left(\sqrt{1+x} - \sqrt{x} \right)^\alpha dx, \quad \alpha > 0; \quad [\text{converge} \Leftrightarrow \alpha > 2]$
16. $\int_{\frac{\pi}{2}}^{+\infty} \frac{\sqrt{|\cos x|}}{\left(x - \frac{\pi}{2}\right)^\alpha \left(x^{\frac{3}{2}} + \frac{3}{2}\right)} dx, \quad \alpha \in \mathbb{R}; \quad [\text{converge} \Leftrightarrow -\frac{1}{2} < \alpha < \frac{3}{2}]$
17. $\int_0^1 \sin(\sqrt{x}) \left(\frac{1-x^2}{x} \right)^\alpha dx, \quad \alpha \in \mathbb{R}; \quad [\text{converge} \Leftrightarrow -1 < \alpha < \frac{3}{2}]$
18. $\int_2^{+\infty} \frac{e^{\frac{1}{x^\alpha}}}{(x^3 - 8)^\alpha} dx, \quad \alpha > 0; \quad [\text{converge} \Leftrightarrow \frac{1}{3} < \alpha < 1]$
19. $\int_0^{+\infty} \frac{|x - \sin x|}{(x + x^3)^\alpha} dx, \quad \alpha > 0; \quad [\text{converge} \Leftrightarrow \frac{2}{3} < \alpha < 4]$
20. $\int_0^{+\infty} \frac{\sqrt{x} \ln(1+x^\alpha)}{1+x^2} dx, \quad \alpha > 0; \quad [\text{converge} \forall \alpha > 0]$
21. $\int_0^1 \frac{1}{\left(\sqrt{x(1+x)} - \sqrt{x}\right)^\alpha} dx, \quad \alpha > 0; \quad [\text{converge} \Leftrightarrow 0 < \alpha < \frac{2}{3}]$
22. $\int_1^{+\infty} (x+1)^{\frac{3}{2}} (e^{\frac{1}{x^2}} - 1)^\alpha dx, \quad \alpha > 0; \quad [\text{converge} \Leftrightarrow \alpha > \frac{5}{4}]$
23. $\int_0^{+\infty} \frac{\ln^4(1+x) e^{\alpha x^3}}{(e^{x^3} - 1)^\alpha (1+x)^\alpha} dx, \quad \alpha > 0; \quad [\text{converge} \Leftrightarrow 1 < \alpha < \frac{5}{3}]$
24. $\int_0^{+\infty} \frac{(e^x - 1)^\alpha}{e^{3x} \arctan(\sqrt[3]{x})(x^2 + x^3)} dx, \quad \alpha > 0; \quad [\text{converge} \Leftrightarrow \frac{4}{3} < \alpha \leq 3]$
25. $\int_0^{+\infty} \frac{\ln(1+x^\alpha)}{\sinh(x^6)(x^2 + \sqrt[6]{x})} dx, \quad \alpha > 0. \quad [\text{converge} \Leftrightarrow \alpha > \frac{31}{6}]$