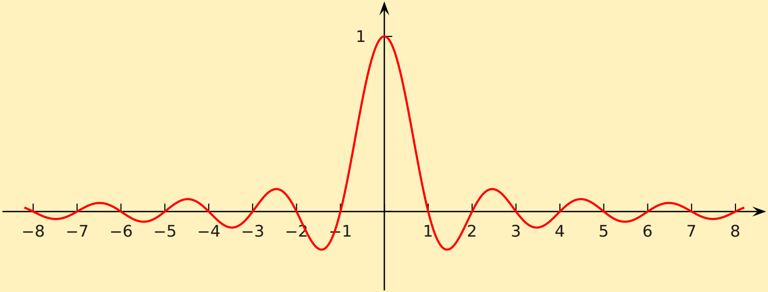
Grafico della funzione sinc.



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Vediamo la funzione approssimante

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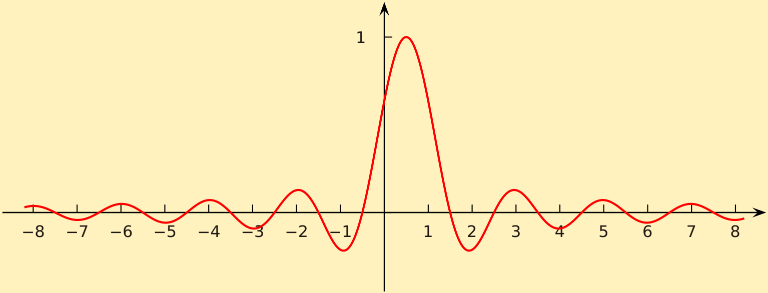

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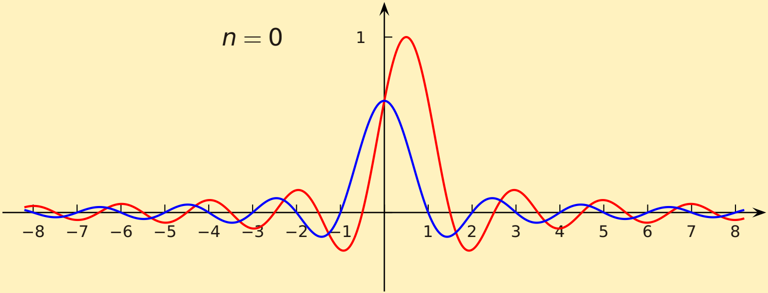

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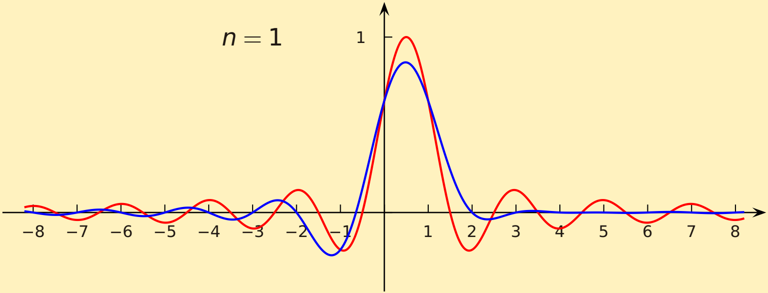

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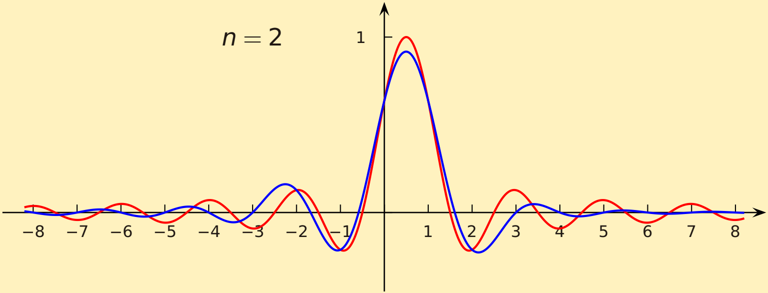

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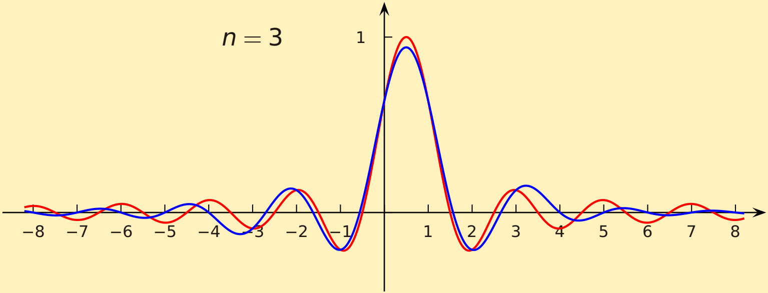

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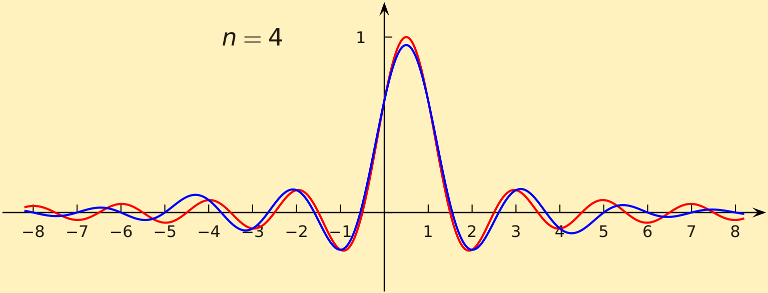

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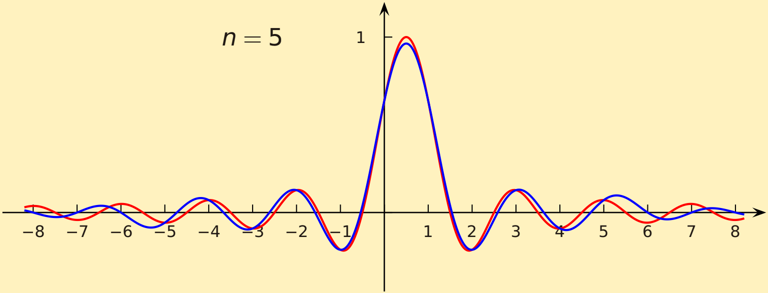

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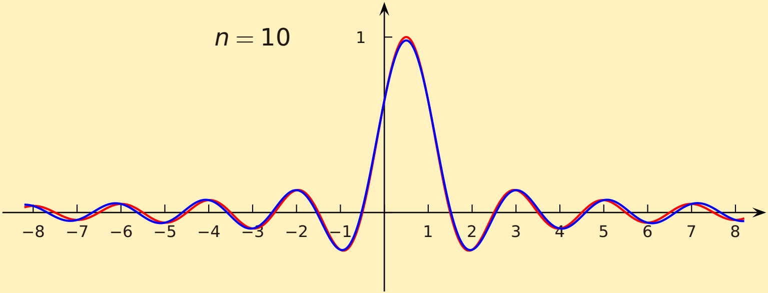

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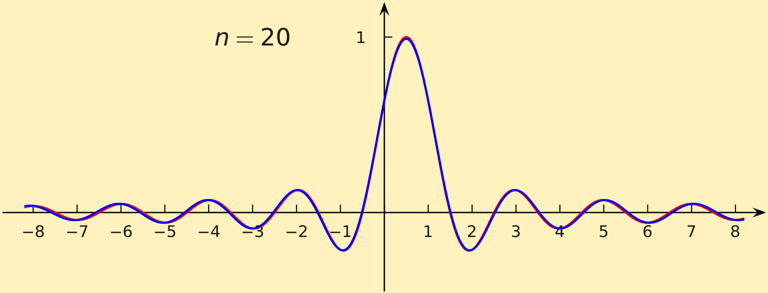

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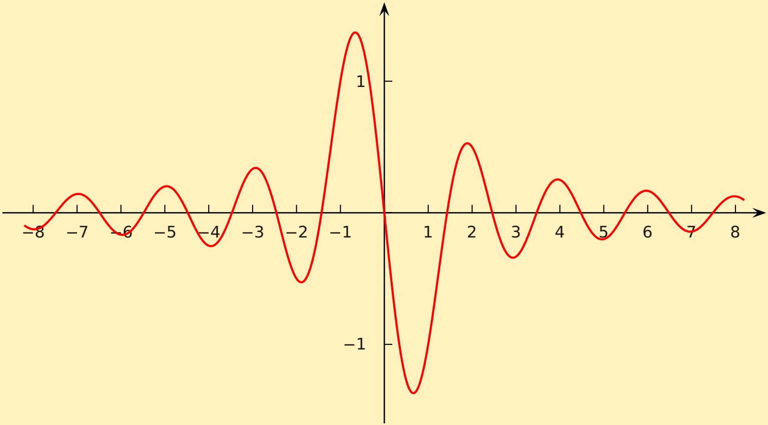

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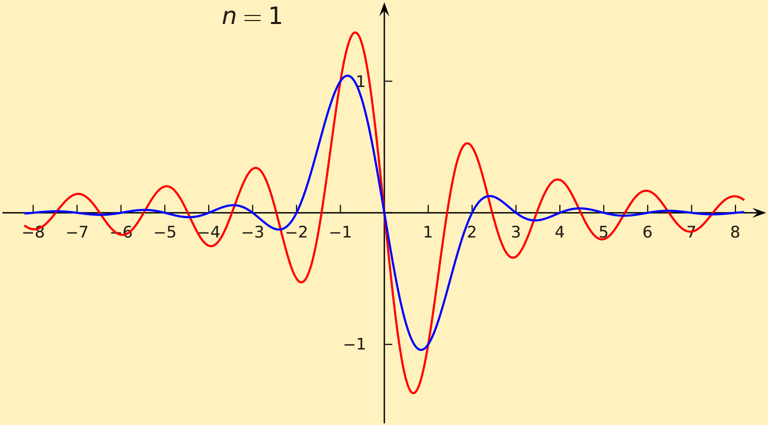

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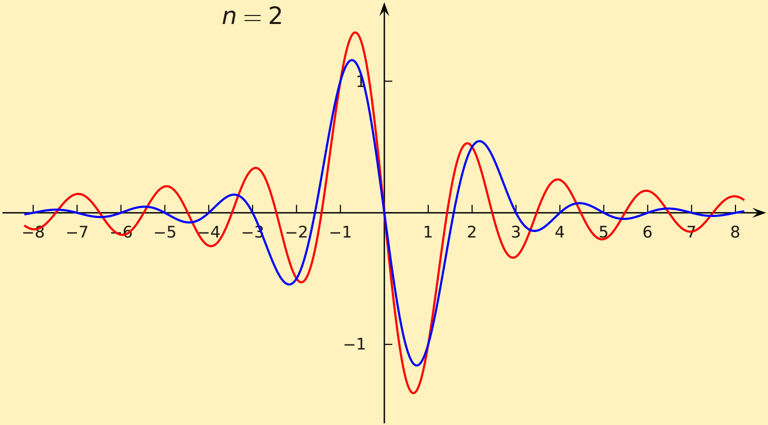

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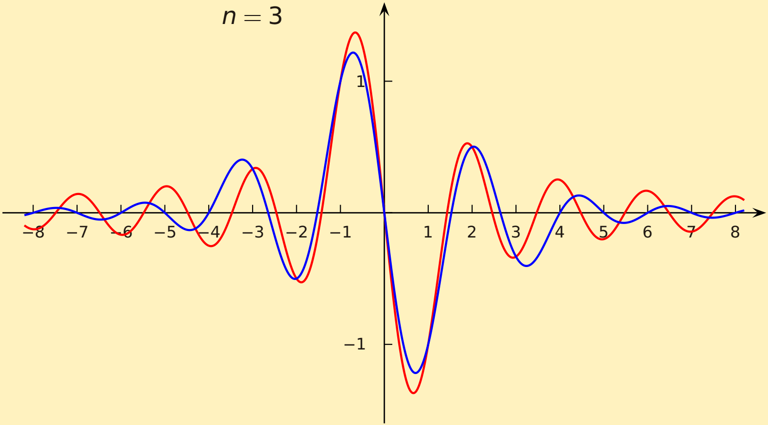

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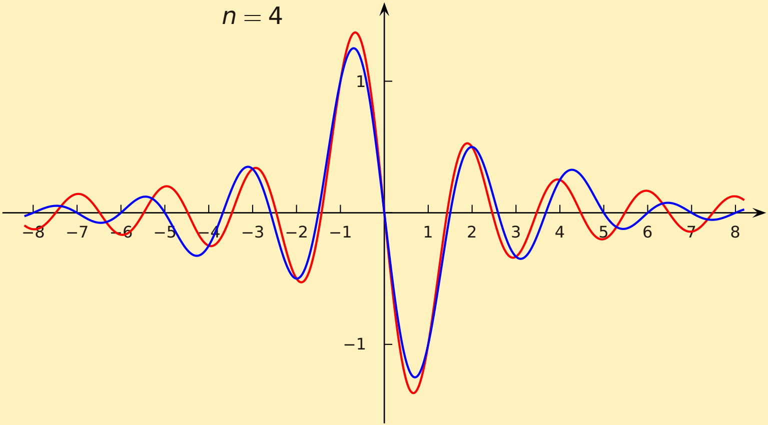

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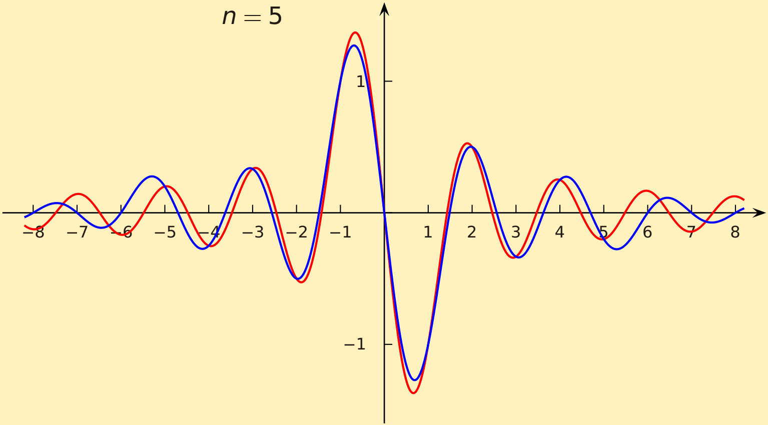

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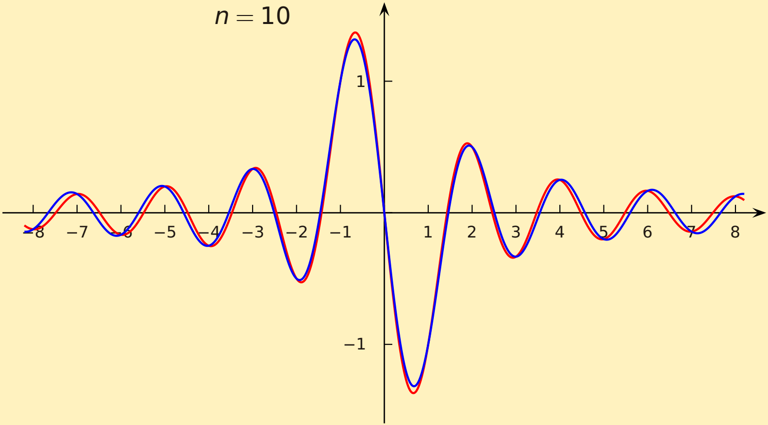

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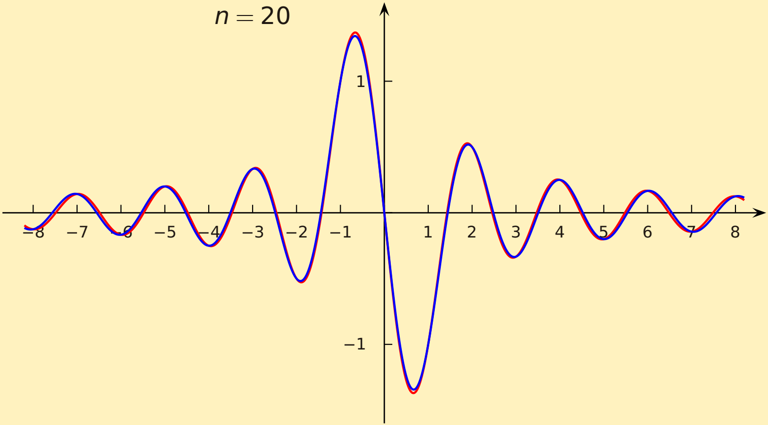

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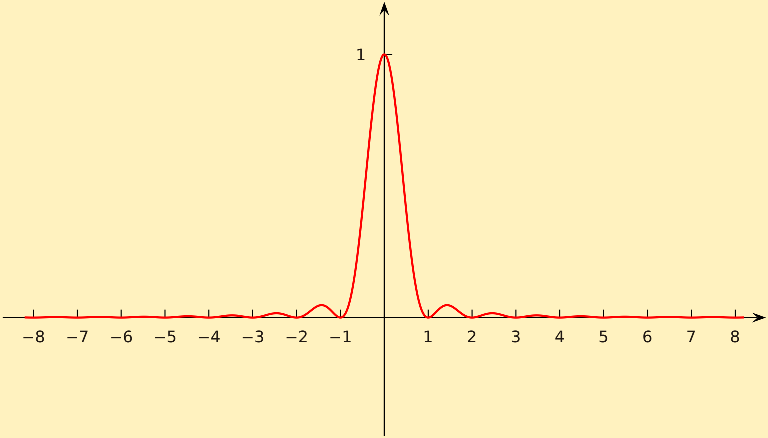
Perciò per il teorema di Shannon

$$\text{sinc}^2(t) = \sum_{k\in\mathbb{Z}} \text{sinc}^2\left(\frac{k}{2}\right)\text{sinc}(2t-k)\,.$$

Osserviamo che

$$\operatorname{sinc}^2\left(\frac{k}{2}\right) = \begin{cases} 1 & \text{per } k = 0 \\ 0 & \text{per } k \neq 0 \text{ pari} \\ \frac{4}{k^2\pi^2} & \text{per } k \text{ dispari} \end{cases}$$

$$x(t) = \mathrm{sinc}^2(t)$$



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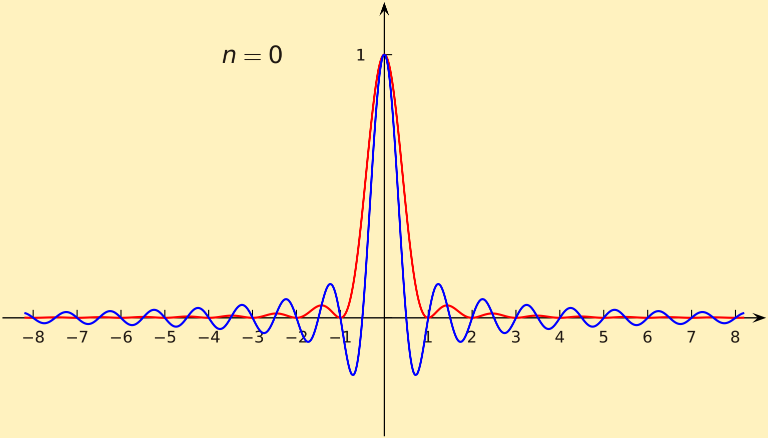

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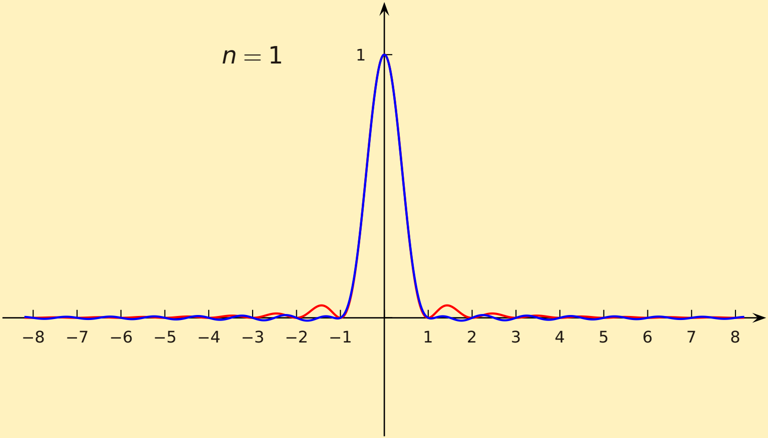

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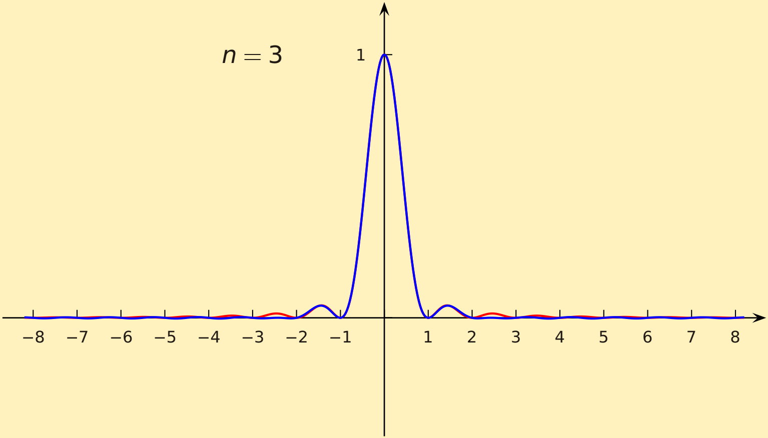

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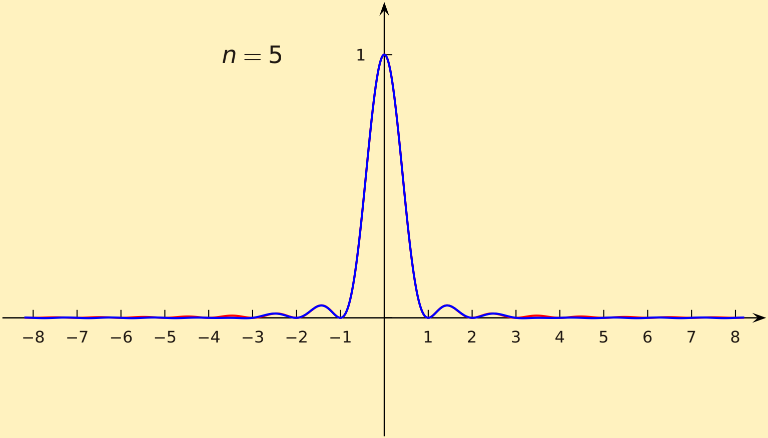

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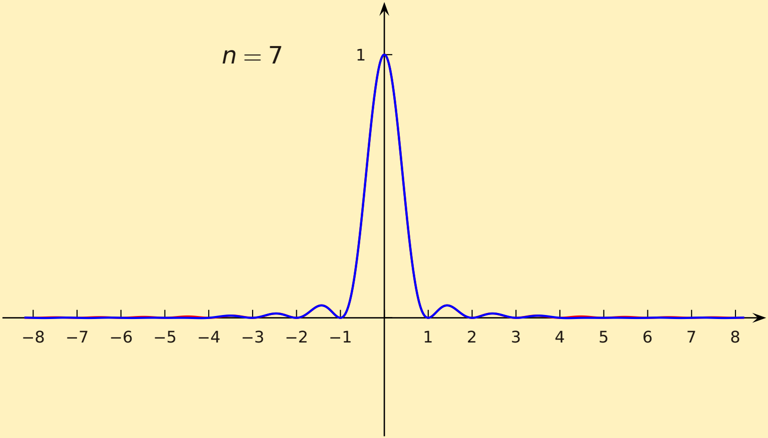

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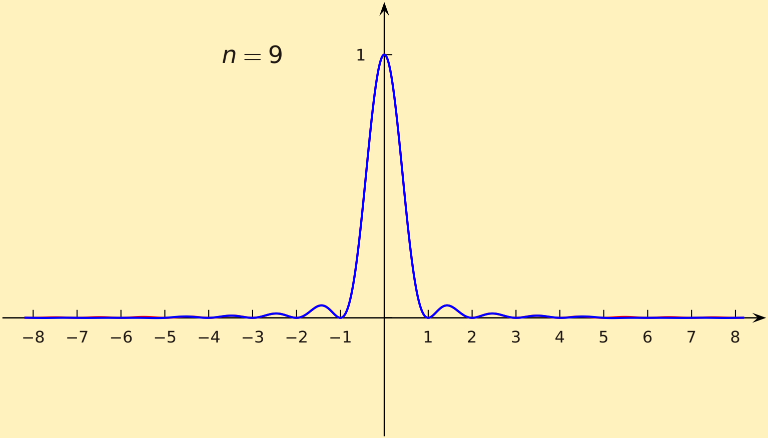

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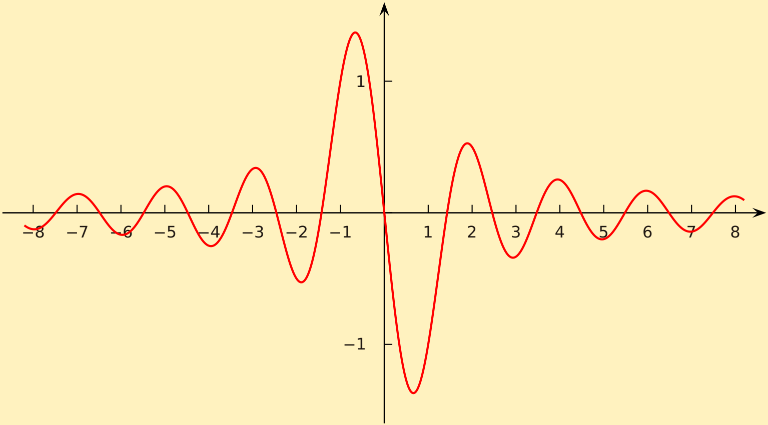

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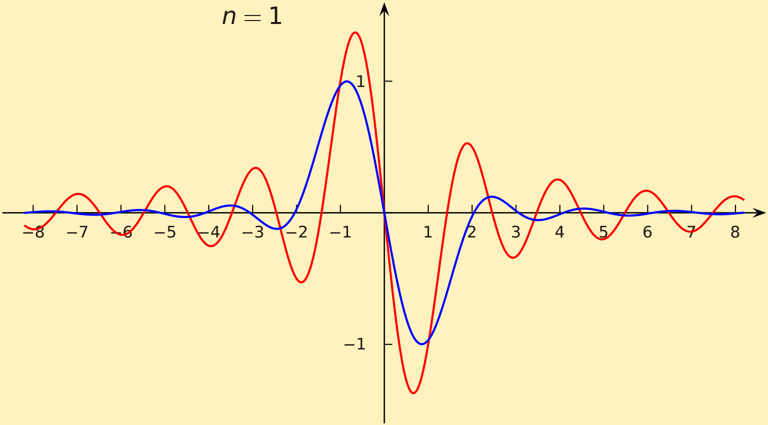

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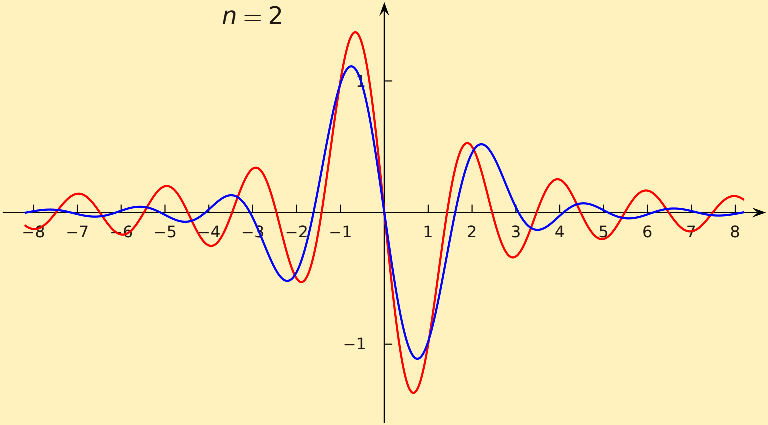

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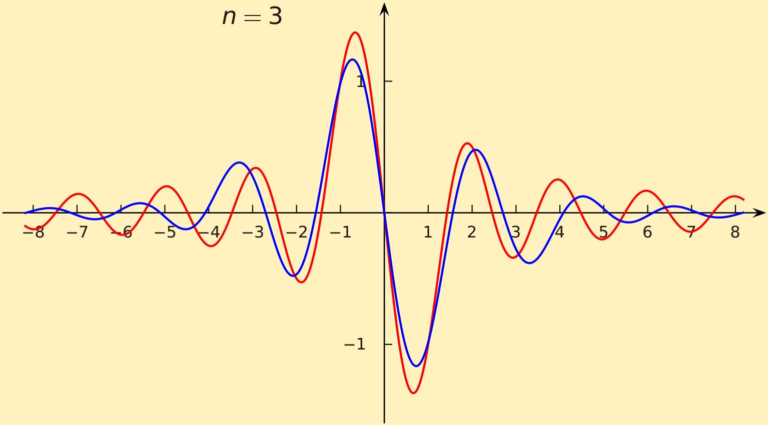

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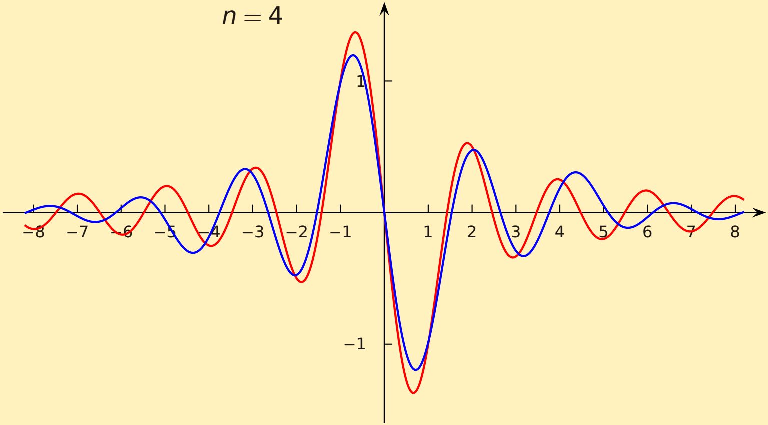

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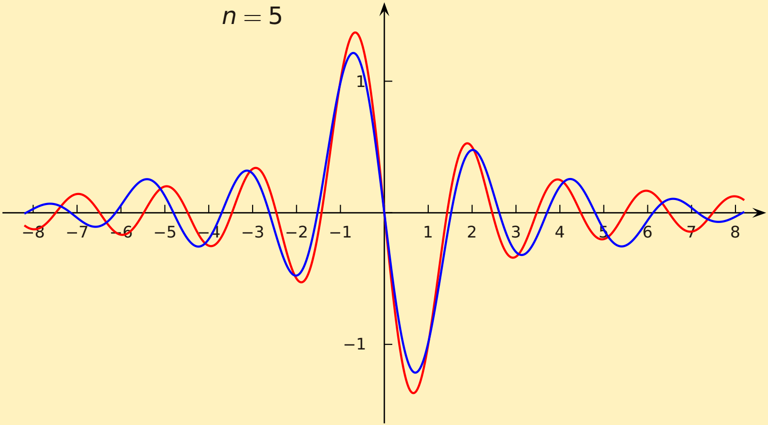

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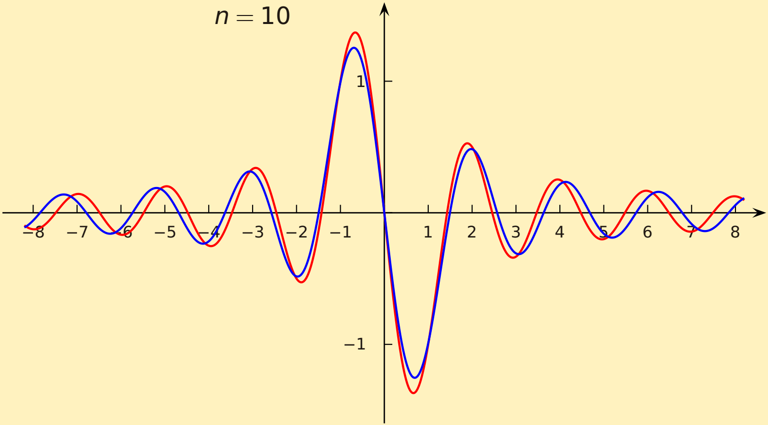

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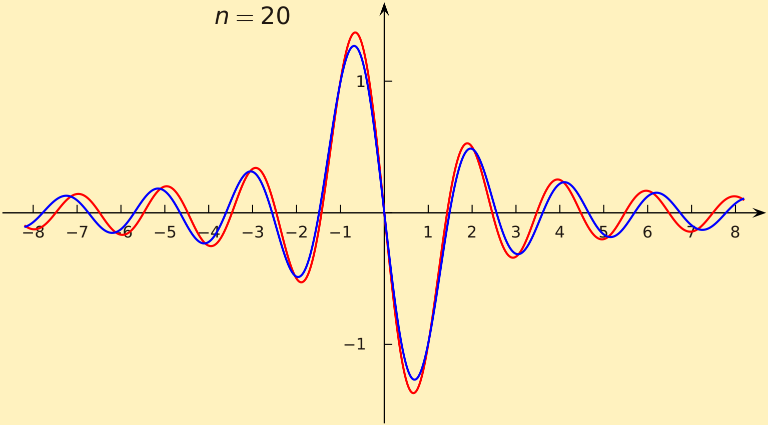

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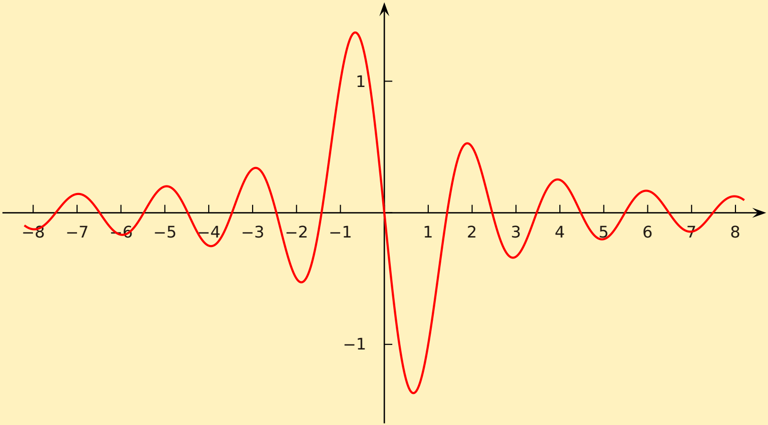

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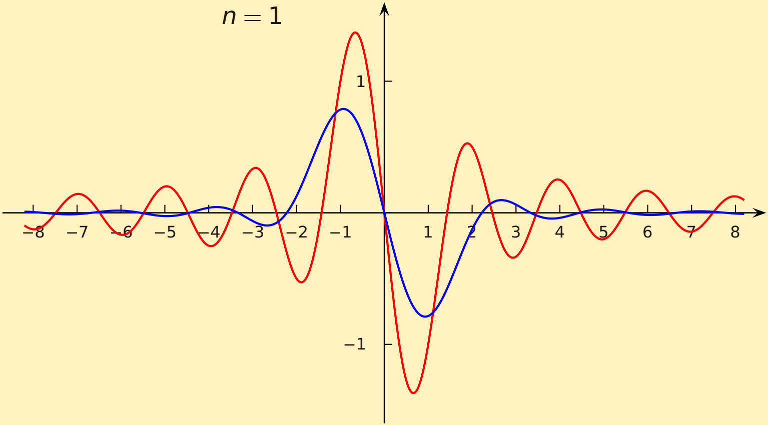
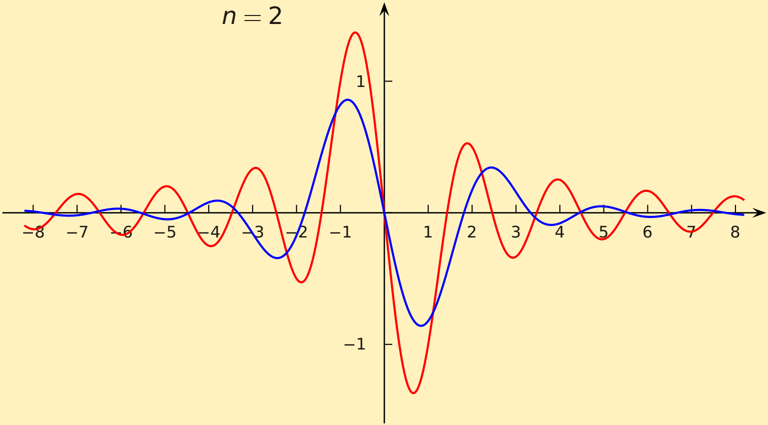
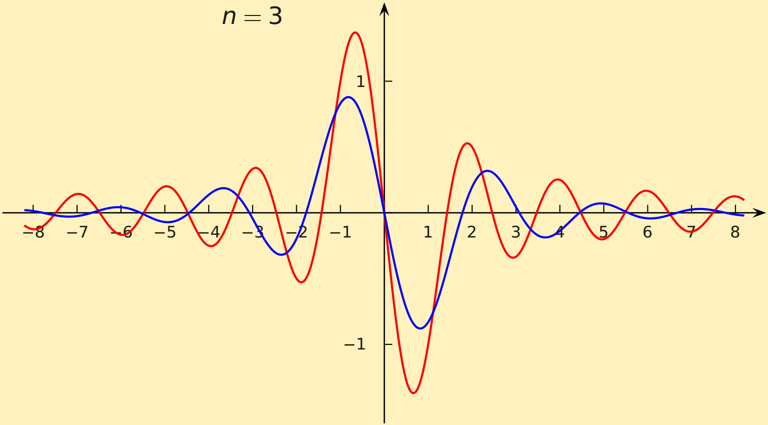


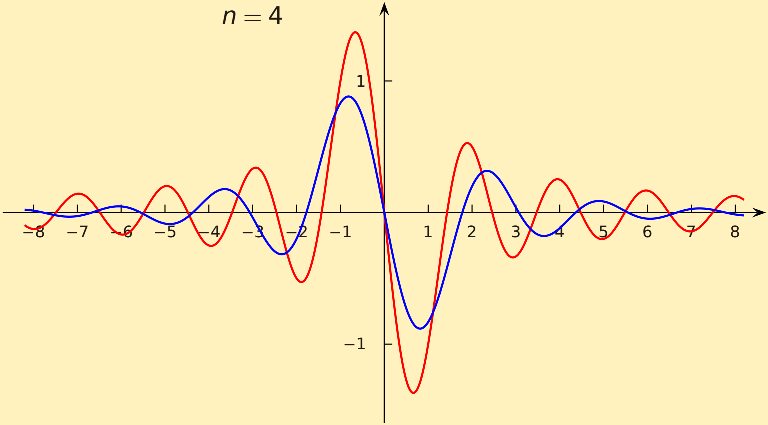
$x(t) = \frac{d}{dt}\,\mathrm{sinc}(t)\,,\ \ a = 0.45\,.$



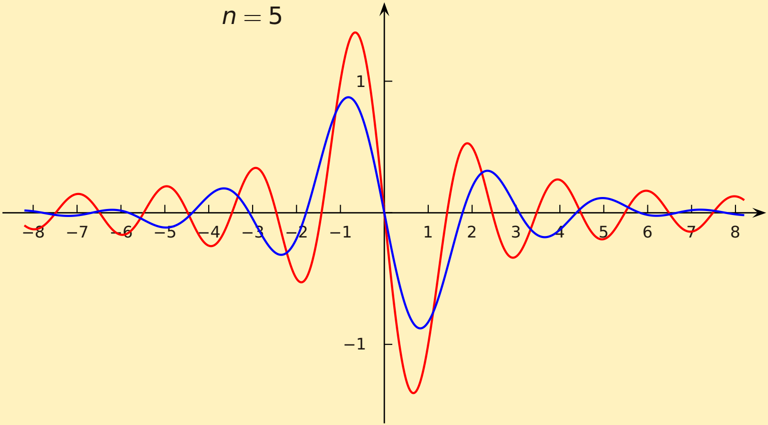
$x(t) = \frac{d}{dt}\,\mathrm{sinc}(t)\,,\ a = 0.45\,.$



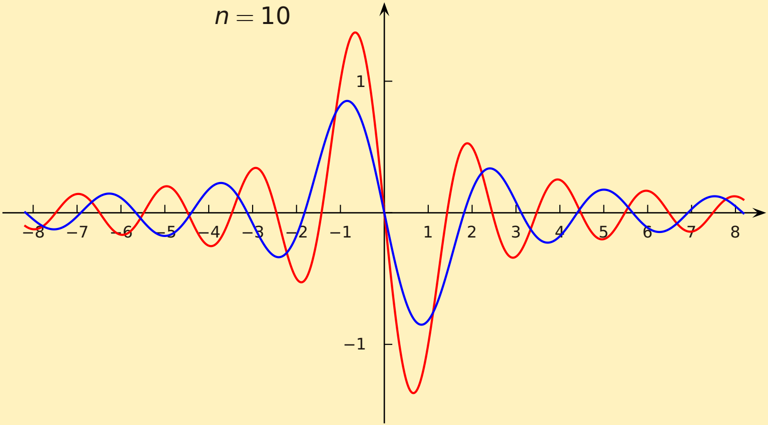
$x(t) = \frac{d}{dt}\,\mathrm{sinc}(t)\,,\ a = 0.45\,.$



$x(t) = \frac{d}{dt}\,\mathrm{sinc}(t)\,,\;\; a = 0.45\,.$



$$x(t) = \frac{d}{dt}\,\mathrm{sinc}(t)\,,\;\; a = 0.45\,.$$



$$x(t) = \frac{d}{dt}\,\mathrm{sinc}(t)\,,\;\; a = 0.45\,.$$