

Corso di Analisi Matematica T-B
Corso di Laurea in Ingegneria Meccanica
Anno Accademico 2012/13

Esercizi

A) Scrivere in forma algebrica i seguenti numeri complessi:

1. $\frac{3-i}{4-i}$

6. $\frac{(2\sqrt{3}+i)^3}{\sqrt{3}-i}$

2. $\frac{2+i}{2i-3}$

7. e^{-2+3i}

3. $\frac{(3+i)^2}{1-i}$

8. $e^{(2+i)^3}$

4. $\frac{2-i}{2+i}$

9. $e^{(1-i)^6}$

5. $\frac{4-3i}{(2+i)^2}$

10. $\frac{e^{2+i}}{e^{3-2i}}$

B) Scrivere in forma trigonometrica i seguenti numeri complessi:

1. $\sqrt{3}-i$

9. $\frac{1-i}{(\sqrt{3}-i)^4}$

2. $-1-3i$

3. $-1+3i$

10. $\frac{(1+2i)(1+i)^8}{(4i)^2}$

4. $\frac{1}{-1+\sqrt{3}i}$

11. $3e^{2-4i}$

5. $\frac{-1+2i}{4i}$

12. $(e^{3-2i})^2$

6. $(-\sqrt{3}+i)^7$

13. $e^{(3-2i)^2}$

7. $(1+4i)^6$

14. $(1-i)e^{2+i}$

8. $\frac{(1+i)^5}{(1-\sqrt{3}i)^3}$

C) Calcolare le radici quadrate e cubiche dei seguenti numeri complessi:

1. -3

4. $1-i$

2. $-i$

5. $-1-2i$

3. $1-\sqrt{3}i$

6. $2+i$

D) Risolvere le seguenti equazioni in campo complesso:

1. $z^2 + z + 8 = 0$

2. $z^2 + iz - 2 = 0$

3. $z^2 + 2z + 1 + 2i = 0$

4. $(3 + 3i)z^2 + \sqrt{5}(2 - 2i)z + 1 + i = 0$

5. $2z^2 + 2(\sqrt{3} + 3i)z - 1 + \sqrt{3}i = 0$

6. $z^2 + 2\sqrt{2}iz - 1 - i = 0$

7. $z^3 + iz = 0$

8. $z^6 - 7z^3 - 8 = 0$

9. $\left(\frac{z-i}{2z+i}\right)^2 = 8i$

10. $\left(\frac{z^2 + 3iz}{z^2 + 2}\right)^2 = 1$

11. $\left(z + \frac{1}{z}\right)^2 = (1-i)^4$

12. $\left(z^2 + \frac{\sqrt{3}}{2} - \frac{1}{2}i\right)^3 = -i$

13. $e^z = -4i$

14. $e^z = -3 + 2i$

15. $e^{iz} = 2 - 2i$

16. $e^{(2+i)z} = 1$

17. $e^{2z} + 6e^z + 9 + 2i = 0$

18. $e^{iz} + 4e^{-iz} = -2$

19. $e^z + e^{-z} = ie^{-z} + i - 2$

20. $(e^{2z} + 4)^3 = (e^{2z} - 4)^3$

Soluzioni

A)

1. $\frac{13}{17} - \frac{1}{17}i$

2. $-\frac{4}{13} - \frac{7}{13}i$

3. $1 + 7i$

4. $\frac{3}{5} - \frac{4}{5}i$

5. $-i$

6. $\frac{19}{4} + \frac{53\sqrt{3}}{4}i$

7. $e^{-2} \cos 3 + i e^{-2} \sin 3$

8. $e^2 \cos 11 + i e^2 \sin 11$

9. $\cos 8 + i \sin 8$

10. $e^{-1} \cos 3 + i e^{-1} \sin 3$

B)

1. $2 \left(\cos\left(\frac{11}{6}\pi\right) + i \sin\left(\frac{11}{6}\pi\right) \right)$

2. $\sqrt{10}(\cos(\arctan 3 + \pi) + i \sin(\arctan 3 + \pi))$

3. $\sqrt{10}(\cos(\pi - \arctan 3) + i \sin(\pi - \arctan 3))$

4. $\frac{1}{2} \left(\cos\left(-\frac{2}{3}\pi\right) + i \sin\left(-\frac{2}{3}\pi\right) \right)$

5. $\frac{\sqrt{5}}{4} \left(\cos\left(\arctan \frac{1}{2}\right) + i \sin\left(\arctan \frac{1}{2}\right) \right)$

6. $128 \left(\cos\left(-\frac{\pi}{6}\right) + i \sin\left(-\frac{\pi}{6}\right) \right)$

7. $17^3(\cos(6 \arctan 4) + i \sin(6 \arctan 4))$

8. $\frac{1}{\sqrt{2}} \left(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4} \right)$

9. $\frac{1}{8\sqrt{2}} \left(\cos\left(\frac{5}{12}\pi\right) + i \sin\left(\frac{5}{12}\pi\right) \right)$

10. $\sqrt{5}(\cos(\pi + \arctan 2) + i \sin(\pi + \arctan 2))$

11. $3e^2(\cos(-4) + i \sin(-4))$

12. $e^6(\cos(-4) + i \sin(-4))$

13. $e^5(\cos(-12) + i \sin(-12))$

14. $e^2 \sqrt{2} \left(\cos\left(1 - \frac{\pi}{4}\right) + i \sin\left(1 - \frac{\pi}{4}\right) \right)$

C)

1. $\pm \sqrt{3}i$;
 $\frac{\sqrt[3]{3}}{2} + \frac{3^{5/6}}{2}i$, $-\sqrt[3]{3}$, $\frac{\sqrt[3]{3}}{2} - \frac{3^{5/6}}{2}i$

2. $\pm \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i \right)$;
 $\frac{\sqrt{3}}{2} - \frac{1}{2}i$, i , $-\frac{\sqrt{3}}{2} - \frac{1}{2}i$

3. $\pm \left(\sqrt{\frac{3}{2}} - \frac{1}{\sqrt{2}}i \right)$;
 $\sqrt[3]{2} \left(\cos\left(\frac{5+6k}{9}\pi\right) + i \sin\left(\frac{5+6k}{9}\pi\right) \right)$,
 $k = 0, 1, 2$

4. $\pm 2^{\frac{1}{4}} \left(\cos\left(\frac{7}{8}\pi\right) + i \sin\left(\frac{7}{8}\pi\right) \right)$;

$2^{\frac{1}{6}} \left(\cos\left(\frac{7}{12}\pi\right) + i \sin\left(\frac{7}{12}\pi\right) \right)$,
 $-\frac{1}{\sqrt[3]{2}} - \frac{1}{\sqrt[3]{2}}i$,

$2^{\frac{1}{6}} \left(\cos\left(\frac{23}{12}\pi\right) + i \sin\left(\frac{23}{12}\pi\right) \right)$

5. $\pm 5^{\frac{1}{4}} \left(\cos \frac{\arctan 2 + \pi}{2} + i \sin \frac{\arctan 2 + \pi}{2} \right)$;

$5^{\frac{1}{6}} \left(\cos \frac{\arctan 2 + k\pi}{3} + i \sin \frac{\arctan 2 + k\pi}{3} \right)$,
 $k = 1, 3, 5$

$$6. \pm 5^{\frac{1}{4}} \left(\cos \frac{\arctan \frac{1}{2}}{2} + i \sin \frac{\arctan \frac{1}{2}}{2} \right);$$

$$5^{\frac{1}{6}} \left(\cos \frac{\arctan \frac{1}{2} + 2k\pi}{3} + i \sin \frac{\arctan \frac{1}{2} + 2k\pi}{3} \right), \\ k = 0, 1, 2$$

D)

$$1. -\frac{1}{2} - \frac{\sqrt{31}}{2}i, -\frac{1}{2} + \frac{\sqrt{31}}{2}i$$

$$2. \frac{\sqrt{7}}{2} - \frac{1}{2}i, -\frac{\sqrt{7}}{2} - \frac{1}{2}i$$

$$3. -2 + i, -i$$

$$4. \frac{\sqrt{5} + 2\sqrt{2}}{3}i, \frac{\sqrt{5} - 2\sqrt{2}}{3}i$$

$$5. \frac{\sqrt{2} - \sqrt{3}}{2} + \frac{\sqrt{6} - 3}{2}i, \\ -\frac{\sqrt{2} + \sqrt{3}}{2} - \frac{\sqrt{6} + 3}{2}i$$

$$6. 2^{1/4} \cos\left(\frac{3}{8}\pi\right) + i \left(-\sqrt{2} + 2^{1/4} \sin\left(\frac{3}{8}\pi\right) \right), \\ -2^{1/4} \cos\left(\frac{3}{8}\pi\right) + i \left(-\sqrt{2} - 2^{1/4} \sin\left(\frac{3}{8}\pi\right) \right)$$

$$7. 0, \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}i, -\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$$

$$8. -1, \frac{1}{2} + \frac{\sqrt{3}}{2}i, \frac{1}{2} - \frac{\sqrt{3}}{2}i, 2, \\ -1 + \sqrt{3}i, -1 - \sqrt{3}i$$

$$9. \frac{6}{41} - \frac{13}{41}i, -\frac{6}{25} - \frac{17}{25}i$$

$$10. -2i, \frac{1}{2}i, -\frac{2}{3}i$$

$$11. \pm(\sqrt{2} + 1)i, \pm(\sqrt{2} - 1)i$$

$$12. 0, \pm \frac{\sqrt[4]{3}}{\sqrt{2}}i, \pm \left(\frac{3^{1/4}}{2} + \frac{3^{3/4}}{2}i \right)$$

$$13. \log 4 + \left(2k - \frac{1}{2} \right) \pi i, k \in \mathbb{Z}$$

$$14. \frac{\log 13}{2} + \left((2k + 1)\pi - \arctan \frac{2}{3} \right) i, k \in \mathbb{Z}$$

$$15. \left(2k - \frac{1}{4} \right) \pi - \frac{\log 8}{2} i, k \in \mathbb{Z}$$

$$16. \frac{2k}{5} \pi + \frac{4k}{5} \pi i, k \in \mathbb{Z}$$

$$17. \frac{\log 5}{2} + \left(\arctan \frac{1}{2} + (2k + 1)\pi \right) i, \\ \frac{\log 17}{2} + \left(-\arctan \frac{1}{4} + (2k + 1)\pi \right) i, k \in \mathbb{Z}$$

$$18. \frac{2}{3} \pi + 2k\pi - \log 2 i, -\frac{2}{3} \pi + 2k\pi - \log 2 i, \\ k \in \mathbb{Z}$$

$$19. (2k + 1)\pi i, \frac{\log 2}{2} + \left(\frac{3}{4} + 2k \right) \pi i, k \in \mathbb{Z}$$

$$20. \frac{1}{2} \log \frac{4}{\sqrt{3}} + \frac{1 + 2k}{4} \pi i, k \in \mathbb{Z}$$