

Corso di Analisi Matematica T-B
Corso di Laurea in Ingegneria Meccanica
Anno Accademico 2012/13

Esercizi

A) Trovare l'integrale generale di:

1. $y'(t) + t^2 y(t) = 0$

3. $y'(t) + \frac{2}{t} y(t) = -t$

2. $y'(t) - \frac{1}{t} y(t) = 2t^2$

4. $y'(t) + \tan t y(t) = 0$

B) Risolvere i seguenti problemi di Cauchy:

1.
$$\begin{cases} y'(t) - \frac{2}{t} y(t) = -\frac{2}{t} \\ y(3) = 2 \end{cases}$$

3.
$$\begin{cases} y'(t) - 3t^2 y(t) = -4t^2 \\ y(0) = 1 \end{cases}$$

2.
$$\begin{cases} y'(t) + \cot t y(t) = 1 \\ y\left(\frac{\pi}{4}\right) = 1 \end{cases}$$

4.
$$\begin{cases} y'(t) - 2ty(t) = 3e^{t^2} \\ y(2) = 0 \end{cases}$$

C) Trovare l'integrale generale di:

1. $y''(t) - 2y'(t) + y(t) = 5t$

4. $y''(t) + 3y'(t) - 4y(t) = \sin t$

2. $y''(t) + 2y'(t) + 2y(t) = 10 \sin t$

5. $y''(t) + 2y'(t) - 3y(t) = te^t$

3. $y''(t) + 2y'(t) = 2$

6. $y''(t) - 4y'(t) + 4y(t) = t^2$

D) Risolvere i seguenti problemi di Cauchy:

1.
$$\begin{cases} y''(t) + 5y'(t) + 6y(t) = 0 \\ y(0) = 1 \\ y'(0) = -1 \end{cases}$$

4.
$$\begin{cases} y''(t) - 6y'(t) + 9y(t) = 12te^{3t} \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$$

2.
$$\begin{cases} y''(t) - 4y'(t) + 4y(t) = 0 \\ y(0) = 2 \\ y'(0) = 1 \end{cases}$$

5.
$$\begin{cases} y''(t) - 2y'(t) + 2y(t) = t \\ y(0) = 0 \\ y'(0) = 0 \end{cases}$$

3.
$$\begin{cases} y''(t) - 2y'(t) + 5y(t) = 0 \\ y(0) = 2 \\ y'(0) = 0 \end{cases}$$

6.
$$\begin{cases} y''(t) - 3y'(t) = 9t^2 - 2 \\ y(0) = 1 \\ y'(0) = 3 \end{cases}$$

E) Trovare l'integrale generale di:

1. $y'''(t) + 2y''(t) - 7y'(t) + 4y(t) = 5e^{-4t}$

3. $y'''(t) + 6y''(t) + 9y'(t) = 3t$

2. $y'''(t) - y''(t) + 4y'(t) - 4y(t) = 5e^t$

4. $y'''(x) - 4y''(x) = 4t + 1$

F) Risolvere i seguenti problemi di Cauchy:

1.
$$\begin{cases} y'''(t) - 4y''(t) + 4y'(t) = 0 \\ y(0) = 1 \\ y'(0) = 2 \\ y''(0) = 3 \end{cases}$$

3.
$$\begin{cases} y'''(t) - y''(t) - 4y'(t) + 4y(t) = e^t \\ y(0) = 0 \\ y'(0) = 0 \\ y''(0) = 0 \end{cases}$$

2.
$$\begin{cases} y'''(t) - 6y''(t) + 11y'(t) - 6y(t) = 0 \\ y(0) = 1 \\ y'(0) = -1 \\ y''(0) = -1 \end{cases}$$

4.
$$\begin{cases} y'''(t) - y''(t) + y'(t) - y(t) = 1 \\ y(0) = 1 \\ y'(0) = 1 \\ y''(0) = 1 \end{cases}$$

G) Risolvere i seguenti problemi di Cauchy:

1.
$$\begin{cases} y'(t) = t(y(t))^3 \\ y(0) = 1 \end{cases}$$

6.
$$\begin{cases} y'(t) = \frac{\sqrt{9 + (y(t))^2}}{\sqrt{\pi^2 + t}} \\ y(0) = 0 \end{cases}$$

2.
$$\begin{cases} y'(t) = \frac{(y(t))^2 - 2}{2ty(t)} \\ y(1) = 1 \end{cases}$$

7.
$$\begin{cases} y'(t) = \frac{\sqrt{9 - (y(t))^2}}{\sqrt{\pi^2 + t}} \\ y(0) = 0 \end{cases}$$

3.
$$\begin{cases} y'(x) = (y(t) + 3) \tan t \\ y\left(\frac{\pi}{3}\right) = -1 \end{cases}$$

8.
$$\begin{cases} y'(x) = \frac{(y(t))^2 + 9}{2t\sqrt{t} + 2\sqrt{t}} \\ y(1) = -3 \end{cases}$$

4.
$$\begin{cases} y'(x) = \frac{(y(t))^2 + 12}{2t^2 + 6} \\ y(3) = 6 \end{cases}$$

9.
$$\begin{cases} y'(t) = \frac{(y(t))^2 + 1}{e^{-t} + 1} \\ y(0) = 1 \end{cases}$$

5.
$$\begin{cases} y'(t) = \frac{t\sqrt{1+t^2}}{(y(t))^2} \\ y(1) = \sqrt{2} \end{cases}$$

10.
$$\begin{cases} y'(t) = \frac{(y(t))^2 - 4}{t - 1} \\ y(0) = 0 \end{cases}$$

Soluzioni

A)

1. $\left\{ ce^{-t^3/3} \mid c \in \mathbb{R} \right\}$

3. $\left\{ ct^{-2} - \frac{1}{4}t^2 \mid c \in \mathbb{R} \right\}$

2. $\left\{ ct + t^3 \mid c \in \mathbb{R} \right\}$

4. $\left\{ c \cos t \mid c \in \mathbb{R} \right\}$

B)

1. $1 + \frac{1}{9}t^2$

3. $\frac{4}{3} - \frac{1}{3}e^{t^3}$

2. $\frac{-\cos t + \sqrt{2}}{\sin t}$

4. $(3t - 6)e^{t^2}$

C)

1. $\left\{ c_1e^t + c_2te^t + 5t + 10 \mid c_1, c_2 \in \mathbb{R} \right\}$

2. $\left\{ c_1e^{-t} \sin t + c_2e^{-t} \cos t + 2 \sin t - 4 \cos t \mid c_1, c_2 \in \mathbb{R} \right\}$

3. $\left\{ c_1 + c_2e^{-2t} + t \mid c_1, c_2 \in \mathbb{R} \right\}$

4. $\left\{ c_1e^t + c_2e^{-4t} - \frac{3}{34} \cos t - \frac{5}{34} \sin t \mid c_1, c_2 \in \mathbb{R} \right\}$

5. $\left\{ c_1e^t + c_2e^{-3t} - \frac{1}{16}te^t + \frac{1}{8}t^2e^t \mid c_1, c_2 \in \mathbb{R} \right\}$

6. $\left\{ c_1e^{2t} + c_2te^{2t} + \frac{1}{4}t^2 + \frac{1}{2}t + \frac{3}{8} \mid c_1, c_2 \in \mathbb{R} \right\}$

D)

1. $2e^{-2t} - e^{-3t}$

4. $2t^3e^{3t}$

2. $2e^{2t} - 3te^{2t}$

5. $-\frac{1}{2}e^t \cos t + \frac{1}{2}t + \frac{1}{2}$

3. $-e^t \sin(2t) + 2e^t \cos(2t)$

6. $e^{3t} - t^3 - t^2$

E)

1. $\left\{ c_1e^t + c_2te^t + c_3e^{-4t} + \frac{1}{5}te^{-4t} \mid c_1, c_2, c_3 \in \mathbb{R} \right\}$

2. $\left\{ c_1e^t + c_2 \sin(2t) + c_3 \cos(2t) + te^t \mid c_1, c_2, c_3 \in \mathbb{R} \right\}$

3. $\left\{ c_1 + c_2e^{-3t} + c_3te^{-3t} + \frac{1}{6}t^2 - \frac{2}{9}t \mid c_1, c_2, c_3 \in \mathbb{R} \right\}$

4. $\left\{ c_1 + c_2t + c_3e^{4t} - \frac{1}{6}t^3 - \frac{1}{4}t^2 \mid c_1, c_2, c_3 \in \mathbb{R} \right\}$

F)

1. $-\frac{1}{4} + \frac{5}{4}e^{2t} - \frac{1}{2}te^{2t}$

2. $5e^t - 6e^{2t} + 2e^{3t}$

3. $-\frac{2}{9}e^t - \frac{1}{3}te^t + \frac{1}{4}e^{2t} - \frac{1}{36}e^{-2t}$

4. $-1 + \frac{3}{2}e^t - \frac{1}{2}\sin t + \frac{1}{2}\cos t$

G)

1. $\frac{1}{\sqrt{1-t^2}}, t \in]-1, 1[$

2. $\sqrt{2-t}, t \in]0, 2[$

3. $-3 + \frac{1}{\cos t}, t \in \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$

4. $2t, t \in \mathbb{R}$

5. $\sqrt{1+t^2}, t \in \mathbb{R}$

6. $3\sinh\left(2\sqrt{\pi^2+t}-2\pi\right), t \in]-\pi^2, +\infty[$

7. $3\sin\left(2\sqrt{\pi^2+t}\right), t \in \left[-\frac{7}{16}\pi^2, \frac{9}{16}\pi^2\right]$

8. $3\tan(3\arctan\sqrt{t}), t \in \left]\frac{1}{3}, +\infty\right[$

9. $\tan\left(\log\frac{e^t+1}{2} + \frac{\pi}{4}\right),$
 $t \in]-\infty, \log(2e^{\pi/4}-1)[$

10. $\frac{2-2(t-1)^4}{(t-1)^4+1}, t \in]-\infty, 1[$