Corso di Analisi Matematica T-B
Corso di Laurea in Ingegneria Meccanica
Anno Accademico 2012/13

Esercizi

A) Calcolare il gradiente delle seguenti funzioni:

1. $f(x,y) = 3\,x^4y^2 - xy$

2. $f(x,y) = \dfrac{xy+y^2}{x-y}$

3. $f(x,y) = e^{x^2} + y\sqrt{xy}$

4. $f(x,y) = \dfrac{xy}{2x+3y}$

5. $f(x,y) = x\,e^y + y^2 \sin x$

6. $f(x,y,z) = xy\,e^{yz}$

7. $f(x,y,z) = xe^{xy+yz}$

8. $f(x,y,z) = \log\dfrac{x+y}{x+z}$

9. $f(x,y,z,t) = xt - yz$

B) Calcolare la matrice jacobiana delle seguenti funzioni:

1. $f(x,y) = (xy, x, y)$

2. $f(x,y) = (3, y\,e^x, x\,e^y)$

3. $f(x,y,z) = (xy+z, xz+y, yz+x)$

4. $f(x,y,z) = \left(xy^2z^3, \dfrac{z^2}{x+y}\right)$

C) Calcolare il gradiente delle seguenti funzioni nel punto indicato:

1. $f(x,y,z) = x^2+y^2 \qquad (1,2,4)$

2. $f(x,y,z) = xy^z \qquad (2,1,3)$

3. $f(x,y) = \dfrac{1}{\sqrt{x^2+y^2}} \qquad (0,-1)$

4. $f(x,y) = \dfrac{x^2+y^3}{x^2+y^2} \qquad (1,0)$

5. $f(x,y,z) = xy^3z - x^2 \qquad (-1,3,0)$

6. $f(x,y) = (\cos x)^{y^2} \qquad \left(\dfrac{\pi}{4}, 2\right)$

D) Calcolare la matrice jacobiana delle seguenti funzioni nel punto indicato:

1. $f(x,y) = (x\cos y, y\sin x) \qquad \left(\pi, \dfrac{\pi}{2}\right)$

2. $f(x,y) = (xy, x+y, x^2y^2) \qquad (-1,2)$

3. $f(x,y) = \left(ye^{x+y}, \log(x^2+y)\right) \qquad (0,1)$

4. $f(x,y,z) = \left(\sqrt{xz}, \sqrt{xyz}\right) \qquad (-2,1,-2)$

5. $f(x,y,z) = (z\sin(xy), x\,e^{yz}) \qquad (\pi,1,2)$

6. $f(x,y,z) = (y\,e^{xz}, xy\,e^z) \qquad (0,1,2)$

E) Calcolare la matrice hessiana delle seguenti funzioni:

1. $f(x,y) = x^2y + x^3 + y$

2. $f(x,y) = x\,e^y$

3. $f(x,y) = x\,e^{x+y}$

4. $f(x,y) = xy + x^2y^2$

5. $f(x,y,z) = xy + 2\,xz + 3\,yz$

6. $f(x,y,z) = x^2 - xz + 3\,yz - z^2$

7. $f(x,y,z) = xyz$

8. $f(x,y,z) = z^2\log(xy)$

Soluzioni

A)

1. $(12\,x^3y^2 - y, 6\,x^4y - x)$

2. $\left(\dfrac{-2\,y^2}{(x-y)^2}, \dfrac{x^2+2\,xy-y^2}{(x-y)^2}\right)$

3. $\left(2xe^{x^2} + \dfrac{y^2}{2\,\sqrt{xy}}, \dfrac{3}{2}\,\sqrt{xy}\right)$

4. $\left(\dfrac{3y^2}{(2x+3y)^2}, \dfrac{2x^2}{(2x+3y)^2}\right)$

5. $(e^y + y^2\cos x, x\,e^y + 2y\sin x)$

6. $\left(y\,e^{yz}, (x+xyz)e^{yz}, xy^2\,e^{yz}\right)$

7. $\left((1+xy)e^{xy+yz}, x(x+z)e^{xy+yz}, xye^{xy+yz}\right)$

8. $\left(\dfrac{z-y}{(x+y)(x+z)}, \dfrac{1}{x+y}, -\dfrac{1}{x+z}\right)$

9. $(t, -z, -y, x)$

B)

1. $\begin{pmatrix} y & x \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$

2. $\begin{pmatrix} 0 & 0 \\ y\,e^x & e^x \\ e^y & x\,e^y \end{pmatrix}$

3. $\begin{pmatrix} y & x & 1 \\ z & 1 & x \\ 1 & z & y \end{pmatrix}$

4. $\begin{pmatrix} y^2z^3 & 2\,xyz^3 & 3\,xy^2z^2 \\ -\dfrac{z^2}{(x+y)^2} & -\dfrac{z^2}{(x+y)^2} & \dfrac{z}{x+y} \end{pmatrix}$

C)

1. $(2, 4, 0)$

2. $(1, 6, 0)$

3. $(0, 1)$

4. $(0, 0)$

5. $(2, 0, -27)$

6. $\left(-1, -\log\sqrt{2}\,\right)$

D)

1. $\begin{pmatrix} 0 & -\pi \\ -\dfrac{\pi}{2} & 0 \end{pmatrix}$

2. $\begin{pmatrix} 2 & -1 \\ 1 & 1 \\ -8 & 4 \end{pmatrix}$

3. $\begin{pmatrix} e & 2e \\ 0 & 1 \end{pmatrix}$

4. $\begin{pmatrix} -\dfrac{1}{2} & 0 & -\dfrac{1}{2} \\ -\dfrac{1}{2} & 1 & -\dfrac{1}{2} \end{pmatrix}$

5. $\begin{pmatrix} -2 & -2\pi & 0 \\ e^2 & 2\pi e^2 & \pi e^2 \end{pmatrix}$

6. $\begin{pmatrix} 2 & 1 & 0 \\ e^2 & 0 & 0 \end{pmatrix}$

E)

1. $\begin{pmatrix} 2\,y + 6\,x & 2\,x \\ 2\,x & 0 \end{pmatrix}$

2. $\begin{pmatrix} 0 & e^y \\ e^y & x\,e^y \end{pmatrix}$

3. $\begin{pmatrix} (x+2)e^{x+y} & (x+1)e^{x+y} \\ (x+1)e^{x+y} & xe^{x+y} \end{pmatrix}$

4. $\begin{pmatrix} 2\,y^2 & 1+4\,xy \\ 1+4\,xy & 2\,x^2 \end{pmatrix}$

5. $\begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 2 & 3 & 0 \end{pmatrix}$

6. $\begin{pmatrix} 2 & 0 & -1 \\ 0 & 0 & 3 \\ -1 & 3 & -2 \end{pmatrix}$

7. $\begin{pmatrix} 0 & z & y \\ z & 0 & x \\ y & x & 0 \end{pmatrix}$

8. $\begin{pmatrix} -\dfrac{z^2}{x^2} & 0 & \dfrac{2z}{x} \\ 0 & -\dfrac{z^2}{y^2} & \dfrac{2z}{y} \\ \dfrac{2z}{x} & \dfrac{2z}{y} & 2\log(xy) \end{pmatrix}$